Sobolev Institute of Mathematics
Novosibirsk State University

Graphs and Groups, Spectra and Symmetries

Novosibirsk, Russia, August, 15 – 28, 2016

Abstracts

Novosibirsk – 2016

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General Information

The International Conference and PhD-Master Summer School on "Graphs and Groups, Spectra and Symmetries"(G2S2) was held on August 15 - 28, 2016, in Novosibirsk, Akademgorodok, Russia. The main goal of this event was to bring together young researchers and famous mathematicians in the field of graph theory and group theory, especially those involving group actions on combinatorial objects.

G2S2 was organized by Sobolev Institute of Mathematics and Novosibirsk State University, Novosibirsk, Russia.

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Website:
math.nsc.ru/conference/g2/g2s2

Conference Program

All scientific activities were held in the Novosibirsk State University, Pirogova 1, Conference room 3307

Monday, August 15

09:00 - 18:00	Registration: *Hall of the Conference room 3307*
19:00 - 21:00	Welcome party: *Hall of the Conference room 3307*

Tuesday, August 16

07:30 - 09:45	*Breakfast*
	PhD-Master Summer School: Minicourse 1
10:00 - 10:50	Lih Hsing Hsu: *Lecture 1*
11:00 - 11:50	Lih Hsing Hsu: *Lecture 2*
11:50 - 12:10	*Coffee break*
	PhD-Master Summer School: Minicourse 2
12:10 - 13:00	Edward Dobson: *Lecture 1*
13:00 - 14:30	*Lunch*
14:30 - 15:20	Edward Dobson: *Lecture 2*
15:20 - 16:00	*Coffee break*
	Conference: Contributed talks
16:00 - 16:25	Sergey Goryainov: *New construction of Deza graphs*
16:30 - 16:55	Leonid Shalaginov: *On Deza graphs with disconnected second neighbourhoods of vertices*
17:00 - 17:25	Vladislav Kabanov: *On Deza graphs with parameters (v, k, k-1, a)*
17:30 - 17:50	Anastasiya Mityanina: *On claw-free strictly Deza graphs*
17:50 - 18:10	*Coffee break*
18:10 - 18:30	Ludmila Tsiovkina: *Arc-transitive antipodal distance-regular covers of complete graphs: almost simple case*
18:35 - 19:00	Stefan Gyürki: *A construction of infinite families of directed strongly regular graphs*
19:00 - 20:00	*Dinner*
20:00 - 22:00	**Problem solving: Minicourse 1**

Wednesday, August 17

07:30 - 09:45	*Breakfast*
	PhD-Master Summer School: Minicourse 1
10:00 - 10:50	Lih Hsing Hsu: *Lecture 3*
11:00 - 11:50	Lih Hsing Hsu: *Lecture 4*
11:50 - 12:10	*Coffee break*
	PhD-Master Summer School: Minicourse 2
12:10 - 13:00	Edward Dobson: *Lecture 3*
13:00 - 14:30	*Lunch*
14:30 - 15:20	Edward Dobson: *Lecture 4*
15:20 - 16:00	*Coffee break*
	Conference: Contributed talks
16:00 - 16:25	Anatoly Kondrat'ev: *Finite groups whose prime graphs do not contain triangles*
16:30 - 16:55	Natalia Maslova: *On realizability of some graphs as Gruenberg–Kegel graphs of finite groups*
17:00 - 17:25	Maria Zvezdina: *On the spectra of automorphic extensions of finite simple exceptional groups of Lie type*
17:30 - 17:50	Viktor Zenkov: *A criterion of unbalance of some simple groups of Lie type*
17:50 - 18:10	*Coffee break*
18:10 - 18:30	Anton Baykalov: *Intersection of conjugate solvable subgroups in classical groups of Lie type*
18:35 - 19:00	Modjtaba Ghorbani: *On the spectra of non-commuting graphs*
19:00 - 20:00	*Dinner*
20:00 - 22:00	**Problem solving: Minicourse 2**

Thursday, August 18

07:30 - 09:45	*Breakfast*
	Conference: Invited Talks
10:00 - 10:50	Dragan Marušič: *On even-closedness of vertex-transitive graphs*
11:00 - 11:50	Klavdija Kutnar: *On colour-preserving automorphisms of Cayley graphs*
11:50 - 12:10	*Coffee break*
12:10 - 13:00	Mikhail Muzychuk: *Isomorphism problem for Cayley combinatorial objects*
13:00 - 14:30	*Lunch*
	Conference: Invited Talks
14:30 - 15:20	Yaokun Wu: *The lit-only σ-game and some mathematics around*
15:20 - 16:00	*Coffee break*
16:00 - 16:50	Tatsuro Ito: *Towards the classication of (P and Q)-polynomial association schemes*
17:00 - 17:50	Jack Koolen: *Applications of Hoffman graphs*
17:50 - 18:10	*Coffee break*
18:10 - 19:00	Alexander Gavrilyuk: *On characterization of the Grassmann graphs $J_2(2d+2,d)$*
19:00 - 20:00	*Dinner*
20:00 - 22:00	**Basketball/Volleyball, NSU Sports Complex**

Friday, August 19

07:30 - 09:45	*Breakfast*
	PhD-Master Summer School: Minicourse 1
10:00 - 10:50	Lih Hsing Hsu: *Lecture 5*
11:00 - 11:50	Lih Hsing Hsu: *Lecture 6*
11:50 - 12:10	*Coffee break*
	PhD-Master Summer School: Minicourse 2
12:10 - 13:00	Edward Dobson: *Lecture 5*
13:00 - 14:30	*Lunch*
14:30 - 15:20	Edward Dobson: *Lecture 6*
15:20 - 16:00	*Coffee break*
	Conference: Contributed talks
16:00 - 16:25	Farzaneh Gholaminezhad: *Structure and Automorphism group of Involution G-Graphs and Cayley Graphs*
16:30 - 16:55	Mina RajabiParsa: *On normal edge-transitive Cayley graphs*
17:00 - 17:25	Swamy Narayan: *Some Class of golden graphs and its construction*
17:30 - 17:50	Anna Simonova: *Small cycles in the Bubble-Sort graph*
17:50 - 18:10	*Coffee break*
18:10 - 18:30	Hamid Reza Golmohammadi: *Improving some bounds for multiple domination parameters in graphs*
18:35 - 19:00	Nikolai Minigulov: *On* 3*-generated lattices with standard and dual standard elements among generators*
19:00 - 20:00	*Dinner*
20:00 - 22:00	**Problem solving: Minicourse 1**

Saturday, August 20

07:30 - 09:45	*Breakfast*
	PhD-Master Summer School: Minicourse 1
10:00 - 10:50	Lih Hsing Hsu: *Lecture 7*
11:00 - 11:50	Lih Hsing Hsu: *Lecture 8*
11:50 - 12:10	*Coffee break*
	PhD-Master Summer School: Minicourse 2
12:10 - 13:00	Edward Dobson: *Lecture 7*
13:00 - 14:30	*Lunch*
14:30 - 15:20	Edward Dobson: *Lecture 8*
15:20 - 16:00	*Coffee break*
	Conference: Contributed talks
16:00 - 16:25	Sho Kubota: *Strongly regular graphs with the same parameters as the symplectic graph*
16:30 - 16:55	Ivan Mogilnykh: *Propelinear codes from multiplicative group of* $GF(2^m)$
17:00 - 17:25	Anastasiya Gorodilova: *The linear spectrum of a quadratic APN function and related open problems*
17:30 - 17:50	Vladimir Potapov: *On the number of* n*-ary quasigroups, Latin hypercubes and MDS codes*
17:50 - 18:10	*Coffee break*
18:10 - 18:30	Anastasia Vasil'eva: *On Fourier decomposition of Preparata-like codes in the graph of the hypercube*
18:35 - 19:00	Anna Taranenko: *On transversals in completely reducible quasigroups and in quasigroups of order 4*
19:00 - 20:00	*Dinner*
20:00 - 22:00	**Problem solving: Minicourse 2**

Sunday, August 21

07:30 - 09:45	*Breakfast*
	Conference: Invited Talks
10:00 - 10:50	Akihiro Munemasa: *Triply even codes obtained from some graphs and finite geometries*
11:00 - 11:50	Patric Östergård: *Is there a* $(4, 27, 2)$ *partial geometry?*
11:50 - 12:10	*Coffee break*
12:10 - 13:00	Lev Kazarin: *Group factorizations, graphs and characters of groups*
13:00 - 14:30	*Lunch*
	Conference: Invited Talks
14:30 - 15:20	Andrey Vasil'ev: *Cartan coherent configurations*
15:20 - 16:00	*Coffee break*
	Conference: Special session
16:00 - 17:00	Ilia Ponomarenko: *Graph isomorphism in quasipolynomial time* (*L. Babai,* 2015)
17:00 - 19:00	Discussions
19:00 - 22:00	*Conference dinner*

Monday, August 22

Excursions/ Sport Activities

Tuesday, August 23

07:30 - 09:45	*Breakfast*
	PhD-Master Summer School: Minicourse 3
10:00 - 10:50	Alexander A. Ivanov: *Lecture 1*
11:00 - 11:50	Alexander A. Ivanov: *Lecture 2*
11:50 - 12:10	*Coffee break*
	PhD-Master Summer School: Minicourse 4
12:10 - 13:00	Bojan Mohar: *Lecture 1*
13:00 - 14:30	*Lunch*
14:30 - 15:20	Bojan Mohar: *Lecture 2*
15:20 - 16:00	*Coffee break*
	Conference: Contributed talks
16:00 - 16:25	Madeleine Whybrow: *Majorana Representations of Triangle-Point Groups*
16:30 - 16:55	Dmitry Churikov: *Automorphism groups of cyclotomic schemes over finite near-fields*
17:00 - 17:25	Olga Kravtsova: *The structure of Hentzel–Rúa semifield of order* 64
17:30 - 17:50	Ilya Matkin: *New infinite family of Cameron-Liebler line classes*
17:50 - 18:10	*Coffee break*
18:10 - 18:30	Ruslan Skuratovskii: *Minimal generating systems and structure of Sylow 2-subgroups of alternating groups* $Syl_2 A_{2^k}$ *and* $Syl_2 A_n$
18:35 - 19:00	Alexey Shlepkin: *About group density function*
19:00 - 20:00	*Dinner*
20:00 - 22:00	**Problem solving: Minicourse 3**

Wednesday, August 24

07:30 - 09:45	*Breakfast*
	PhD-Master Summer School: Minicourse 3
10:00 - 10:50	Alexander A. Ivanov: *Lecture 3*
11:00 - 11:50	Alexander A. Ivanov: *Lecture 4*
11:50 - 12:10	*Coffee break*
	PhD-Master Summer School: Minicourse 4
12:10 - 13:00	Bojan Mohar: *Lecture 3*
13:00 - 14:30	*Lunch*
14:30 - 15:20	Bojan Mohar: *Lecture 4*
15:20 - 16:00	*Coffee break*
	Conference: Contributed talks
16:00 - 16:25	Shuchita Goyal: *Hom complex of Mapping cylinders of graphs*
16:30 - 16:55	Mukesh Kumar Nagar: *A* q *and* q,t*-analogue of Hook Immanantal Inequalities and Hadamard Inequality for Trees*
17:00 - 17:25	Maryam Jalali-Rad: *Erdös-Ko-Rado Properties of some Finite Groups*
17:30 - 17:50	Roman Panenko: Φ*-Harmonic Functions on Graphs*
17:50 - 18:10	*Coffee break*
18:10 - 18:30	Keiji Ito: *Maximum skew energy of tournaments*
18:35 - 19:00	Michele Mulazzani: 4*-colored graphs and complements of knots and links*
19:00 - 20:00	*Dinner*
20:00 - 22:00	**Problem solving: Minicourse 4**

Thursday, August 25

07:30 - 09:45	*Breakfast*
	Conference: Invited Talks
10:00 - 10:50	Anton Betten: *Graphs with Integral Spectrum*
11:00 - 11:50	Yuriy Tarannikov: *On plateaued Boolean functions with the same spectrum support*
11:50 - 12:10	*Coffee break*
12:10 - 13:00	Jin Ho Kwak: *Notes on zeta functions of regular graphs*
13:00 - 14:30	*Lunch*
	Conference: Invited Talks
14:30 - 15:20	Shaofei Du: 2-*Arc-Transitive Regular Covers*
15:20 - 16:00	*Coffee break*
16:00 - 16:50	Ilia Ponomarenko: *On characterizations of association schemes by intersection numbers*
17:00 - 17:50	Matan Ziv-Av: *A family of regular coherent non-Schurian graphs, related to extremal graph theory*
17:50 - 18:10	*Coffee break*
18:10 - 20:00	**Conference: Open problems session**
20:00 - 22:00	**Basketball/Volleyball, NSU Sports Complex**

Friday, August 26

08:30 - 09:45	*Breakfast*
	PhD-Master Summer School: Minicourse 3
10:00 - 10:50	Alexander A. Ivanov: *Lecture 5*
11:00 - 11:50	Alexander A. Ivanov: *Lecture 6*
11:50 - 12:10	*Coffee break*
	PhD-Master Summer School: Minicourse 4
12:10 - 13:00	Bojan Mohar: *Lecture 5*
13:00 - 14:30	*Lunch*
14:30 - 15:20	Bojan Mohar: *Lecture 6*
15:20 - 16:00	*Coffee break*
	Conference: Contributed talks
16:00 - 16:25	Grigory Ryabov: *On the isomorphism problem for Cayley graphs over abelian p-groups*
16:30 - 16:55	Sven Reichard: *Schur rings over elementary abelian two-groups*
17:00 - 17:25	Alexandr Valyuzhenich: *Minimal supports of eigenfunctions of Hamming graphs*
17:30 - 17:50	Ilya Mednykh: *Circulant graphs and Jacobians*
17:50 - 18:10	*Coffee break*
18:10 - 18:30	Daria Andryukhina: *Spectra in ensembles of regular graphs*
18:35 - 19:00	Konstantin Kobylkin: *Computational complexity of Vertex Cover and related problems for highly connected*
19:00 - 20:00	*Dinner*
20:00 - 22:00	**Problem solving: Minicourse 3**

Saturday, August 27

08:30 - 09:45	*Breakfast*
	PhD-Master Summer School: Minicourse 3
10:00 - 10:50	Alexander A. Ivanov: *Lecture 7*
11:00 - 11:50	Alexander A. Ivanov: *Lecture 8*
11:50 - 12:10	*Coffee break*
	PhD-Master Summer School: Minicourse 4
12:10 - 13:00	Bojan Mohar: *Lecture 7*
13:00 - 14:30	*Lunch*
14:30 - 15:20	Bojan Mohar: *Lecture 8*
15:20 - 16:00	*Coffee break*
	Conference: Contributed talks
16:00 - 16:25	Yuhei Inoue: *The four color theorem and Thompson's F*
16:30 - 16:55	Aleksey Glebov: *Splitting planar graphs of bounded girth to subgraphs with short paths*
17:00 - 17:25	Igor Bykov: *Completability of hamiltonian cycle in halved cube*
17:30 - 17:50	Fadekemi Janet Osaye: *On the average eccentricities of some forbidden subgraphs*
17:50 - 18:10	*Coffee break*
18:10 - 18:30	Somayeh Heydari: *Some simple groups which are determined by their character degree graphs*
18:35 - 19:00	Irina Starikova: *Mathematical Beauty*
19:00 - 20:00	*Dinner*
20:00 - 22:00	**Problem solving: Minicourse 4**

Sunday, August 28

08:30 - 09:45	*Breakfast*
	Conference: Invited Talks
10:00 - 10:50	Mitsugu Hirasaka: *Congruence of triangles in a metric space*
11:00 - 11:50	Ji-Young Ham: *On the volume and the Chern-Simons invariant for the 2-bridge link orbifolds*
11:50 - 12:10	*Coffee break*
12:10 - 13:00	Roman Nedela: *Hamilton Cycles in Graphs Embedded into Surfaces*
	Closing

Abstracts

Abstracts of Minicourses, Plenary and Contributed talks are listed alphabetically with respect to the Presenting Author

Minicourses

Minicourse I: Another viewpoint of Euler graphs and Hamiltonian graphs

Lecturer:
Lih-Hsing Hsu
Distinguished Professor, Providence University, Taichung, Taiwan
lihhsing@gmail.com

It may appear that there is little left to do in regards to the study of the Hamiltonian property of vertex transitive graphs unless there is a major breakthrough on the famous Lovasz conjecture. However, if we extend the concept of the traditional Hamiltonian property to other Hamiltonicity properties, then there is still much left to explore. In this series of lectures, I will introduce some of these Hamiltonicity properties, namely fault tolerant Hamiltonian, spanning connectivity, and mutually independent Hamiltonicity.

The course contains eight lectures.

Minicourse II: The Cayley Isomorphism Problem

Lecturer:
Edward Dobson
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University of Primorska, Koper, Slovenia
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In 1967 Ádám conjectured that two circulant graphs $\mathrm{Cay}(\mathbb{Z}_n, S)$ and $\mathrm{Cay}(\mathbb{Z}_n, T)$ are isomorphic if and only if there exists $m \in \mathbb{Z}_n^*$ such that $mS = T$. While this conjecture is not true (although from two different points of view it is mostly true), the conjecture was quickly generalized to ask for which groups G any two Cayley graphs $\mathrm{Cay}(G, S)$ and $\mathrm{Cay}(G, T)$ are isomorphic if and only if they are isomorphic by an automorphism of G (or $\alpha(S) = T$ for some automorphism $\alpha \in \mathrm{Aut}(G)$). Such a group G is a **CI-group with respect to graphs**. It is easy to show that $\alpha(\mathrm{Cay}(G, T))$ is a Cayley graph of G for every subset T of G and $\alpha \in \mathrm{Aut}(G)$, so in testing isomorphism between two Cayley graphs of a group G one must always check to see if the automorphisms of G are isomorphisms. From this point of view, asking whether or not a group is a CI-group with respect to graphs is the same as asking if the minimal or necessary list of permutations that must be checked as possible isomorphisms is also a sufficient list of permutations to check. We will develop some of the main tools that are used to determine if a group is a CI-group with respect to graphs, along with appropriate permutation group theory. The groups G we will focus on will mainly be of small order (where small order means that there are not many prime factors). These groups are rich enough to illustrate some, but not all, of the proof techniques that have been developed to show a group is a CI-group with respect to graphs as well as to highlight some of the obstacles for a group to be a CI-group with respect to graphs. We will also discuss how the techniques developed to attack the Cayley isomorphism problem can be modified to attack the isomorphism problem from graphs that are highly symmetric but not Cayley graphs nor even vertex-transitive, as well as to attack similar isomorphism problems for other classes of combinatorial objects.

The course contains eight lectures.

Minicourse III: Graphs and their eigenvalues

Lecturer:
Bojan Mohar
Simon Fraser University, Canada
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The course contains eight lectures:

1-2. Adjacency matrix and its eigenvalues (basic properties, Perron–Frobenius theory, interlacing, quotients and equitable partitions, distance-regular graphs). Algebraic and combinatorial properties.
3. Laplacian and expansion (Laplacian matrix, expansion lemma, expanders and Ramanujan graphs)
4-5. Random graphs (random graphs and random matrices, Wigner's semicircle theorem, extensions, quasirandom graphs).
6-7. Applications (Huckel theory, HOMO-LUMO separation, perfect graphs, more on expanders, regularity lemma and graph limits).
8. Hermitian adjacency matrix of a digraph.

Minicourse IV: Y-groups via Majorana Theory

Lecturer:
Alexander A. Ivanov
Department of Mathematics, Imperial College, South Kensington, London, UK
a.ivanov@imperial.ac.uk

Motivated by an earlier observation by B. Fischer, around 1980 J.H. Conway conjectured that a specific Coxeter diagram Y_{443} together with a single additional (so-called "spider") relation form a presentation for the direct product of the largest sporadic simple group known as the Monster and a group of order 2. This conjecture was proved by S.P. Norton and the lecturer in 1990. It appears promising to revisit this subject through currently developing axiomatic approach to the Monster and its non-associative 196884-dimensional algebra, which goes under the name "Majorana Theory".

The course contains eight lectures.

Plenary Talks

Graphs with Integral Spectrum

Anton Betten
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The spectrum of a graph is the set of eigenvalues of the adjacency matrix of the graph, together with their multiplicities. In 1974, Harary and Schwenk initiate the study of graphs with integral spectra, that is, graphs whose eigenvalues are all integral.

In this talk, we will look at integral Cayley graphs and highlight some open problems [1]. One question is whether the Cayley graph obtained from the symmetric group with respect to the generators of the form $(1, i)$ $i = 2, \ldots, n$ is integral. This graph is known as the star graph. A connection to the representation theory of the symmetric group is explored [2].

References

[1] A. Abdollahi, E. Vatandoost, Which Cayley graphs are integral? *Electron. J. Combin.* **16(1)** (2009) 17.

[2] Laszlo Babai, Spectra of Cayley Graphs. *Journal of Combinatorial Theory* **27B** (1979) 180–189.

2-Arc-Transitive Regular Covers

Shaofei Du
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A cover X of a given graph Y is an homomorphism ϕ from X to Y, locally it is a bijection. This is one of fundamental and important concepts in topological graph theory. Another motivation for us to study covers might be from classifications of finite arc-transitive graphs, mainly 2-arc-transitive graphs. In this talk, I shall collect some our recent results on 2-arc-transitive regular covers. In particular, by exhibiting some examples I try to show you the relationships between construction of covers and group extension theory, group representation theory and topological graph theory.

On characterization of the Grassmann graphs $J_2(2d+2, d)$

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Krasovskii Institute of Mathematics and Mechanics UB RAS, Yekaterinburg, Russia
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This is joint work with Jack Koolen

The Grassmann graph $J_q(n, d)$, $n \geq 2d$, is a graph (of diameter d) defined on the set of d-dimensional subspaces of an n-dimensional vector space over the finite field $\mathbb{F}_q$, with two subspaces being adjacent if their intersection has dimension $d-1$.

In 1995, Metsch [1] showed that a distance-regular graph with the same intersection array as $J_q(n, d)$ is indeed $J_q(n, d)$ unless $n = 2d$, $n = 2d+1$, ($n = 2d+2$ if $q \in \{2, 3\}$), or ($n = 2d+3$ if $q = 2$).

In 2005, Van Dam and Koolen [2] constructed the twisted Grassmann graphs, a family of distance-regular graphs with the same intersection array as $J_q(2d+1, d)$, but not isomorphic to them, for all prime powers q and $d \geq 2$.

In 2015, the authors showed that the Grassmann graph $J_2(2d, d)$ can be characterized by its intersection array, if the diameter d is an odd number or large enough.

In this talk, we will discuss a characterization of the Grassmann graphs $J_2(2d+2, d)$.

References

[1] K. Metsch, A characterization of Grassmann graphs. *European J. Combin.* **16** (1995) 171–195.

[2] E. R. van Dam, J. H. Koolen, A new family of distance-regular graphs with unbounded diameter. *Invent. Math.* **162** (2005) 189–193.

On the volume and the Chern-Simons invariant for the 2-bridge link orbifolds

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Department of Mathematical Sciences, Seoul National University, Seoul, Korea
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This is joint work with Joongul Lee, Alexander Mednykh and Aleksey Rasskazov

We extend some part of the unpublished paper [1] written by Mednykh and Rasskazov. Using the approach indicated in this paper we derive the Riley–Mednykh polynomial for some family of the two bridge link orbifolds. As a result we obtain explicit formulae for the volume of cone–manifolds and the Chern–Simons invariant of orbifolds of the knot with Conway's notation $C(2n, 4)$.

References

[1] A. Mednykh, A. Rasskazov, On the structure of the canonical fundamental set for the 2–bridge link orbifolds. (1998).

[2] A. Mednykh, A. Rasskazov, Volumes and degeneration of cone-structures on the figure–eight knot. *Tokyo J. Math.* **29(2)** (2006) 445–464.

[3] Ji-Y. Ham, A. Mednykh, V. Petrov, Trigonometric identities and volumes of the hyperbolic twist knot cone-manifolds. *J. Knot Theory Ramifications* **23(12)** (2014)1450064, 16.

[4] Ji-Y. Ham, J. Lee, Explicit formulae for Chern–Simons invariants of the twist knot orbifolds and edge polynomials of twist knots. *arXiv:1411.2383* (2014). Preprint.

[5] Ji-Y. Ham, J. Lee, The volume of hyperbolic cone–manifolds of the knot with Conway's notation $C(2n, 3)$. *arXiv:1512.05481* (2015). Preprint.

[6] Ji-Y. Ham, J. Lee, Explicit formulae for Chern–Simons invariants of the hyperbolic orbifolds of the knot with Conway's notation $C(2n, 3)$. *arXiv:1601.00723* (2016). Preprint.

Characterization of finite metric spaces by their isometric sequences

Mitsugu Hirasaka
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This is a joint work with Masashi Shinohara

Let (X, d) be a metric space where $d : X \times X \to \mathbb{R}_{\geq 0}$ is a metric function. For $A, B \subseteq X$ we say that A is *isometric* to B if there exists a bijection $f : A \to B$ such that $d(x, y) = d(f(x), f(y))$ for all $x, y \in A$. We shall write $A \simeq B$ if A is isometric to B. For a positive integer k we denote by $A_k(X)$ the quotient set of $\binom{X}{k}$ by $\simeq$, i.e.

$$A_k(X) = \left\{ [A] \mid A \in \binom{X}{k} \right\},$$

where $[A]$ is the isometry class containing A. For a finite metric space (X, d) we call $(|A_i(X)| : i = 1, 2, \ldots, |X|)$ the *isometric sequence* of X. In this talk we aim to characterize metric spaces X by their isometric sequences, and classify them with the property $|A_2(X)| = |A_3(X)| \leq 3$.

Towards the classification of (P and Q)-polynomial association schemes

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This is a survey talk on the subject of the title above. In his lectures at Ohio State University in the late 70s, Eiichi Bannai proposed the classification of (P and Q)-polynomial association schemes; he regarded them as finite, combinatorial analogue of compact symmetric spaces of rank 1. I will trace the history back to the late 60s and explain how the concepts of P/Q-polynomial association schemes arose in relation to finite permutation groups, coding/design theory. I will then overview the progress of the classification in the 80s, 90s and thereafter. Finally I will present my personal view about the scope for the classification problem.

This talk is based on my lecture at the GAP seminar of USTC which is aimed at helping graduate students bridge the gap between established mathematics and the frontiers of mathematical research.

Group factorizations, graphs and characters of groups

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1. Group factorizations. Let G be a group and A, B be its subgroups. The group G has a factorization $G = AB$ if every element $g \in G$ can be expressed in the form $g = ab$ with $a \in A, b \in B$. By famous Burnside's p^{α}-lemma (1903) the group G, having the conjugacy class $\boldsymbol{x^G}$ of a prime-power size $p^{\alpha} > 1$ is non-simple. Clearly, in this case G has a factorization of the form: $G = C_G(x)P$, where P is a Sylow p-subgroup. As an immediate consequence, every group of order $p^a q^b$ for prime numbers p, q and natural numbers a, b is soluble. Further investigations due to H. Wielandt, O. Kegel, B. Huppert, N. Itô and others leads to many classical result in this area. For instance, finite group $G = AB$ with nilpotent subgroups A and B is soluble. The last result in this area, not using FSGC, is a theorem of L. Kazarin (1979), solving Shemetkov - Scott conjecture: the group $G = AB$ factorized by subgroups A and B such that A and B have nilpotent subgroups A_0 and B_0 of index at most 2 in the corresponding group, is soluble.

Later (in 1990) M. Liebeck, Sh. Praeger and J. Saxle have classified maximal factorizations of all finite simple groups, using FSGC. However, many simple problems, concerning factorizations, remains open. A short survey of the results in this area could be find in [1]. Some new results were obtained in this century.

Recall that the group X is called π-decomposable, if G is a direct product of its Hall π-subgroup $O_{\pi}(X)$ and a subgroup $O_{\pi'}(X)$ of coprime order. The following (containing classical results due H. Wielandt and O. Kegel) was proved by L. Kazarin, A. Martinez-Pastor and M. D. Perez-Ramos [2] in 2015.

Theorem 1. *Let π be a set of odd primes. If a finite group $G = AB$ is a product of two π-decomposable subgroups A and B, then $O_{\pi}(A)O_{\pi}(B)$ is a subgroup of G.*

As a corollary, we prove that the product $G = AB = AC = BC$ of permutable finite π-decomposable subgroups A, B and C is π-decomposable.

A generalization of some results due to Z. Arad, E. Fisman and E. M. Palchik is also presented in the talk.

Note that there is a natural "geometric" situation, when the factorizations are raised. If G is a transitive permutation group acting on a set Ω and a subgroup $H \leq G$ also acts transitively on Ω, then G has a factorization $G = HK$, where K is a stabilizer of a point $\alpha \in \Omega$.

There is another type of factorizations. They are, so-called, ABA-factorizations. More precisely, let A and B be a subgroups of G. We say that G is an ABA-group, if for every element $g \in G$ there exist $a, a' \in A$ and $b \in B$ such that $g = aba'$. There are many interesting classes of groups possessing non-trivial ABA-factorizations. Among them all finite simple groups of Lie type and alternating groups of permutation of degree $n \geq 5$. It is unknown whether every sporadic simple group possesses non-trivial ABA-factorization. There are some interesting results about such factorizations since first papers of D. Gorenstein and I. M. Herstein. But in general the situation is very complicated. One recent result [3] belongs to B. Amberg and L. Kazarin:

Theorem 2. *Let a finite group $G = ABA$ cyclic subgroup B. If A is abelian or A is nilpotent of odd order and $GCD(|A|, |B|) = 1$, then G is soluble.*

Note that the structure of a nonsoluble ABA-group with abelian subgroups A and B is still unknown.

Of course, every 2-transitive permutation group is an ABA-group for every subgroup B, not contained in A. It seems that such factorizations exists more often if G is a rank 3 permutation group. In each case the authors [3] have find some new approach to this problem based on the properties of involutions.

2. Some arithmetic properties of the characters of groups. It is well-known that the main tool for the proofs of theorems concerning groups with factorizations was character theory. This is clear for Burnside's p^α-lemma. In general, a finite group has a factorization $G = AB$ iff $\langle 1_A^G, 1_B^G \rangle = 1$. Similar criterions exist also for 2-transitive groups and rank 3 permutation groups.

E. P. Wigner has proved (in 1941) very interesting result, concerning finite groups with the following property. Let G be a real finite group all whose all irreducible representations are $T_1, T_2, \ldots T_k$. If for any $i, j \leq k$ the decomposition $T_i \otimes T_j = \sum c_{ij}^s T_s$ has all coefficients $c_{ij}^s \leq 1$, then the following holds:

$$\sum_{g \in G} |C_g(g)|^2 = \sum_{g \in G} \zeta(g)^3.$$

Here $\zeta(g)$ is the number of solutions in G of the equation $x^2 = g$. E.Wigner called groups with this property SR-groups. The solubility of finite SR-groups was proved by L. Kazarin with his students in 2010. One of the results of similar nature obtained in 2011 with B. Amberg, is as follows:

Theorem 3. *Let G be a finite simple group and τ be an arbitrary involution of G. If $|G| > 2|C_G(\tau)|$, then G has a proper subgroup of order at least $|G|^{1/2}$. If $|G| > |C_G(\tau)|^3$, then $|G| < k(G)^3$, where $k(G)$ is a class number of G.*

The behavior of the degrees of irreducible characters is of special interest for many authors. One of the famous computational problems in computational mathematics is the complexity of matrix computation. In celebrated works of Umans with coauthors this is reduced in some sense to estimate of the number $\sum_{\chi \in Irr(G)} \chi(1)^3$ for certain groups G. The cases when the group G has an irreducible character of large degree, is very interesting. One new result was obtained in [2] by L. Kazarin and S. Poiseeva.

Theorem 4. *Let G be a finite group with an irreducible character Θ such that $|G| \leq 2\Theta(1)^2$. If G is not a 2-group, then every irreducible character of G is a constituent of Θ^2. If $\Theta(1) = pq$ for some primes p and q, then G has an abelian normal subgroup N of index pq.*

There are many simple groups G with the property $|G| < c\chi(1)^2$ for some irreducible character χ of G and some small constant c. As an example, for $c < 3$ there is the Thompson group Th of order 190373967.

3. Graphs on the sets of primes. There are several types of graphs determined on the prime divisors of the order of a group. Let x be a natural number and $\pi(x)$ be the set of its prime divisors. If X is the set of natural numbers, then $\rho(X) = \cup_{x \in X} \pi(x)$. Denote the graph $\Gamma(X)$ with the set $\rho(X) = V(X)$ of its vertices. Two vertices are adjacent if $pq \mid x$ for some $x \in X$.

Another graph $\Delta(X)$ on the set X is defined as follows. Vertices a and b are adjacent, if the greatest common divisor of a and b is bigger than one.

It seems that the first (after Cayley) graphs in group theory were invented by S. A. Chounikhin in 1938. In an explicit form this was done by L. Kazarin in 1978. Prime graph of Grünberg-Kegel, $GK(G)$, became popular since 1981 after paper by J. S. Williams and later by A. S. Kondratiev in connection of a program of characterization of a simple groups by spectrums. In $GK(G)$ the set X is the set of prime divisors of elements of G. In this case primes p and q are adjacent if there exists in G an element whose order is pq.

Another prime graph $\Gamma_{sol}(G)$ was invented by S. Abe and N. Iiyori. In this case X is the set of a prime divisors of soluble subgroups of G. Two primes p and q are adjacent if there exists a soluble subgroup of G whose order is divisible by pq.

One of recent results for this graphs related to finite simple groups is due B. Amberg and L. Kazarin. Previously S. Abe and N. Iiyori described finite simple groups whose graph $\Gamma_{sol}(G)$ is a clique. Define by $t_s(G)$ the largest number of independent vertices in $\Gamma_{sol}(G)$.

Theorem 4. *Let G be a finite simple group such that $t_s(G) = 2$ (i.e. the dual graph to $\Gamma_{sol}(G)$ has no triangles). Then G is isomorphic to one of the following groups: $L_n^{\pm}(q)(n \leq 7), S_4(q), P\Omega_8^+(2), {}^3D_4(2), {}^2F_4(2)', G_2(3), S_6(2), M_{11}, M_{12}, M_{22}, HS, McL, J_2$ or $A_n (n \leq 10)$.*

The proof uses two papers by A. V. Vasiliev and E. P. Vdovin. As a corollary we obtain the description of slightly larger class of finite simple groups, than groups having a factorization by two soluble subgroups.

Theorem 5. *Let G be a finite simple group with soluble subgroups A and B. If $\pi(G) = \pi(A) \cup \pi(B)$, then G belongs to the list of groups in the conclusion of Theorem 4.*

The graph $\Gamma_A(G)$ was defined by L. Kazarin, A. Martinez-Pastor and M. D. Perez-Ramos in 2005. This graph is defined on the set of prime divisors of the order of a group G in a following manner. Two vertices p and q in $\pi(G)$ are adjacent if for a Sylow p subgroup P of G the order of a group $N_G(P)/PC_G(P)$ is divisible by q. Of course, the edge (q, p) exists if $|N_G(Q)/QC_G(Q)|$ is divisible by p.

One of the important results concerning these graphs is as follows:

Theorem 6. *Let G be a finite almost simple group. Then the graph $\Gamma_A(G)$ is connected.*

Note that if (p, q) is an edge in $\Gamma_A(G)$, then (p, q) is an edge in $\Gamma_{sol}(G)$, but the graph $\Gamma_A(G)$ of a soluble group could be non-connected. Hence our theorem 6 gives another proof of a theorem by S. Abe and N. Iiyori. Theorem 6 is a main tool for some results in formation theory concerning formation closed uner taking of normalizers of Sylow subgroups.

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Applications of Hoffman graphs

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In this talk I will discuss the origins and recent applications of Hoffman graphs as defined by Woo and Neumaier (1995). Among these applications are to study graphs with smallest eigenvalue -3, constructing (regular) graphs with a fixed smallest eigenvalue, and trees with spectral radius three.

On colour-preserving automorphisms of Cayley graphs

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This is joint work with Ademir Hujdurović, Edward Dobson, Dave Morris and Joy Morris

We study the automorphisms of a Cayley graph that preserve its natural edge-colouring. In this talk recent results about colour-preserving automorphisms of Cayley graphs will be presented. More precisely, we are interested in groups G, such that every such automorphism of every connected Cayley graph on G has a very simple form: the composition of a left-translation and a group automorphism. We find classes of groups that have the property, and we determine the orders of all groups that do not have the property. We also have analogous results for automorphisms that permute the colours, rather than preserving them.

Notes on zeta functions of regular graphs

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In this talk, we quickly review zeta functions of (di)graphs and computing L-functions associated with some graph coverings.

On even-closedness of vertex-transitive graphs

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When dealing with symmetry properties of combinatorial objects, such as graphs admitting a transitive group action, that is, *vertex-transitive graphs*, one of the fundamental questions is to determine their full automorphism group. While some symmetries of such objects are obvious, certain additional symmetries remain hidden or difficult to grasp. When this is the case, the goal is to find a reason for their existence and a method for describing them.

Along these lines the above question reads as follows: *Given a transitive group H acting on a set V of vertices of a graph, determine whether H is its full automorphism group or not. When the answer is no, find a method to describe the additional automorphisms.*

We propose to study such group "extensions" by considering the existence of *odd automorphisms* (as opposed to even automorphisms) that is automorphisms that act as odd permutations on the vertex set of a graph. The implications go beyond the simplicity of the concept of even/odd permutations alone. For example, given a group H consists of even permutations only, a partial answer to the above question could be given provided the structure of the graph in question forces existence of automorphisms acting as odd automorphisms.

In this talk some recent results in regards to the above problem will be considered. A special emphasis will be given to the class of cubic symmetric graphs where a complete solution will be presented. These results suggest that the even/odd question is likely to uncover certain much more complex structural properties of graphs that go beyond simple arithmetic conditions.

Triply even codes obtained from some graphs and finite geometries

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This is joint work with Koichi Betsumiya

A triply even code is a binary linear code in which the weight of every codeword is divisible by 8. By Lam and Yamauchi [3], every triply even code of length a multiple of 16 containing the all-ones vector is the structure code of some holomorphic framed vertex operator algebra. Motivated by this fact, we classified maximal triply even codes of length 48, and discovered an infinite family can be obtained from the triangular graphs [1].

In this talk, we present another infinite family of triply even codes, derived from the odd-orthogonal graphs [1, Section 12.2]. Let V be a 4-dimensional vector space over a finite field of odd characteristic, equipped with a nondegenerate quadratic form of Witt index 1. Define a graph Γ whose vertex set is the set of nonisotropic projective points of plus type, where two vertices are adjacent whenever the line through these points is a tangent. Then the row vectors of the adjacency matrix of Γ generate a triply even code of length $q(q^2+1)/2$.

The proof of this fact amounts to showing that the number of common neighbors of three distinct vertices is always even.

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Isomorphism problem for Cayley combinatorial objects

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A Cayley object over a finite group H is any relational structure $\mathcal{R}$ with point set H which is invariant under the group of right translations H_R. The well-known examples of Cayley objects include Cayley graphs, Cayley maps, group codes etc. The isomorphism problem for Cayley objects may be formulated as follows: Given two combinatorial objects over the group H, find whether they are isomorphic or not.

In may talk I'll present the old and the new results which solves the above problem for different classes of objects.

Hamilton Cycles in Graphs Embedded into Surfaces

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This is joint work with M. Kotrbčík and M. Škoviera

Although hundreds of papers deals with the problem of existence of a Hamiltonian cycle in a graph, there is a luck of results on the hamiltonicity of cubic graphs. Among others, it is well-known that to decide whether a cubic graph is Hamiltonian is an NP-complete problem. The main idea of the talk is to present a new approach to investigate hamiltonicity of graphs. Instead of graphs, we consider graphs embedded into closed surfaces such that each face is bounded by a circuit (no repetitions of vertices are allowed). Such an embedding is called polytopal or circular. By the cycle-double-cover conjecture every 2-connected graph admits a polytopal embedding. In an embedded graph the set of hamilton cycles split into three classes: contractible, bounding but non-contractible cycles and non-separating cycles. We shall investigate bounding and contractible hamilton cycles.

Assume first that the underlying surface is sphere. Due to the Jordan curve theorem a Hamilton cycle C in a spherical map $\mathcal{M}$ separates the surface into two disks bounded by C. Consider the two disjoint sets of faces $\mathcal{A}$ and $\mathcal{B}$ separated by C. Then it is not difficult to see that the corresponding sets of vertices $\mathcal{A}^*$ and $\mathcal{B}^*$ in the dual map induce two disjoint trees in $\mathcal{M}^*$. The main idea consists in reversing the above process. We shall try to identify a proper tree $T \subseteq \mathcal{M}$ (or a "one-face embedded"subgraph) in the dual $\mathcal{M}^*$, such that the topological closure of the faces in $\mathcal{M}$ corresponding to the vertices of T will form a bordered surface with a connected boundary creating a bounding hamilton cycle in $\mathcal{M}$. We shall call the tree T a co-hamiltonian tree. Now, let $\mathcal{M}$ be a polytopal map. In general, a bounding hamilton cycle in $\mathcal{M}$ will always define a bi-partition of the vertex set of the dual map. In order to understand what sort of bi-partitions give a hamilton cycle in the original map, we introduce a useful concept of a 1-sided subgraph which generalizes the concept of an embedded tree. Using this concept we were able to prove a theorem stating that a map $\mathcal{M}$ on a surface S admits a bounding hamilton cycle if and only if the vertex set of the dual $\mathcal{M}^*$ admits a partition into two subsets which induce one-sided subgraphs H and K such that $\beta(H) + \beta(K) = \epsilon(S)$, where $\beta(H)$ and $\beta(K)$ are the Betti numbers and $\epsilon(S)$ is the Euler genus. The hamilton cycle is contractible if and only if one of the subgraphs H, K is a tree. The two subgraphs satisfying the statement will be called co-hamiltonian subgraphs.

Under certain circumstances we can guarantee existence of a vertex-bipartition in the dual map into two co-hamiltonian subgraphs, or we can prove that such a decomposition cannot exist. For instance, we show that the truncation of a triangulation without a separating 3-cycle has a hamilton path and if the number of triangles is congruent 2 mod 4 it has a bounding hamilton cycle. Also we shall deal with truncations of triangulations with faces of size at most 7. We show that such map is either hamiltonian, if the number of triangles is congruent 2 mod 4, or it has a hamilton path. This relates the result to a conjecture by Barnette, recently proved by Kardoš, stating that cubic polyhedral graphs with faces of size at most six are hamiltonian. Also we present a uniform approach to the problem of hamiltonicity of Cayley graphs coming from groups of the form

$$\langle x, y \mid y^2 = (xy)^3 = 1, \dots \rangle,$$

investigated in papers by Glower, Youngs, Marušič, Kutnar and Malnič. A new result proves hamiltonicity, or at least existence of a hamilton path in Cayley graphs generated by three involutions x, y and z satisfying the relations $(xy)^3 = (yz)^3 = 1$. These are particular instances of a folklore conjecture stating that Cayley graphs are hamiltonian which solution does not seem to be in hand.

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Is there a $(4, 27, 2)$ partial geometry?

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This is joint work with Leonard Soicher

A *partial geometry* with parameters (s, t, α) consists of lines and points with the properties that (i) each line has $s + 1$ points and two distinct lines intersect in at most one point; (ii) each point is on $t + 1$ lines and two distinct point occur on at most one line; and (iii) for each point p that does not lie on a line l, there are exactly α lines through p that intersect l. The question whether there exists a $(4, 27, 2)$ partial geometry has tantalized researchers during the last couple of decades. Such a partial geometry would have 275 points and 1540 lines and its point graph would be a $(275, 112, 30, 56)$ strongly regular graph (srg). There is a unique srg with the aforementioned parameters called the *McLaughlin graph*. In this talk, a computer search for a $(4, 27, 2)$ partial geometry starting from the McLaughlin graph is described. After 270 core-years and more than one physical year, the computers claim that there is no such partial geometry.

On characterizations of association schemes by intersection numbers

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A coherent configuration $\mathcal{X}$ is characterized by the intersection numbers if every algebraic isomorphism of this configuration to another one is induced by a combinatorial isomorphism; in this case, $\mathcal{X}$ is said to be separable. The importance of this notion is explained by the fact that if the coherent configuration of a graph is separable, then the isomorphism of this graph to any other graph can be tested by the Weisfeiler-Leman algorithm [4]. Besides, the separability of a distance-regular graph (or, more general, of an association scheme) means in terms of [1], that the graph is uniquely determined by its parameters.

The index of an association scheme $\mathcal{X}$ with n points and m relations of valency 1 is defined to be the number n/m. In [2,3], it was proved that every quasi-thin or pseudocyclic scheme $\mathcal{X}$ is separable whenever the index of $\mathcal{X}$ is enough large in comparison with its maximal valency. It turns out that a similar result (with much better bound) holds for the class of TI-schemes, which contains the most part of quasi-thin schemes and all pseudocyclic schemes. Here, a TI-scheme can be thought as a combinatorial analog of the coherent configuration of a transitive group G, the point stabilizer of which is a TI-subgroup of G.

As a byproduct of the main result, we prove that every association scheme of prime degree p and valency k is schurian, whenever $p > 1 + 6k(k-1)^2$. This improves [3, Corollary 1.2], where the lower bound for p was $O(k^5)$.

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On plateaued Boolean functions with the same spectrum support

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The Boolean function f is a map $\mathbf{F}_2^n \to \mathbf{F}_2$. The Walsh coefficient $W_f(u)$ (also known as a spectral coefficient), $u \in \mathbf{F}_2^n$, is defined as the real-valued sum:

$$W_f(u) = \sum_{x \in \mathbf{F}_2^n} (-1)^{f(x) + <x,u>}.$$

The plateaued Boolean function is the Boolean function whose Walsh coefficients take values $\{0, \pm 2^c\}$ for some integer c. The given set of Walsh coefficients defines the Boolean function uniquely. If the spectrum support of a plateaued Boolean function f is known (i. e. the set of all vectors $u \in \mathbf{F}_2^n$ such that $W_f(u) \neq 0$) then only signs of all Walsh coefficients are known, so the plateaued Boolean function is not defined uniquely. For the majority of spectrum supports S including the full space $\mathbf{F}_2^n$, n even, $n > 8$, the number of plateaued functions with this spectrum support S is unknown whereas for some specific families the number of functions with such spectrum support was found (see for example [1]). We present some such constructions of spectra and analyse their symmetries. Also we discuss the problem of possible values of a rank (or an affine rank) for given spectrum supports of plateaued Boolean functions. This problem earlier was studied in $[2, 3]$ and recently the new upper bound for an arbitrary Boolean function with the given cardinality of a spectrum support (also known as a sparsity) was obtained in [4].

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Cartan coherent configurations

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This is joint work with Ilia Ponomarenko

The Cartan scheme $\mathcal{X}$ of a finite group G with a (B, N)-pair is defined to be the coherent configuration associated with the action of G on the right cosets of the Cartan subgroup $B \cap N$ by the right multiplications. It is proved that if G is a simple group of Lie type, then asymptotically, the coherent configuration $\mathcal{X}$ is 2-separable, i.e. the array of 2-dimensional intersection numbers determines $\mathcal{X}$ up to isomorphism. It is also proved that in this case, the base number of $\mathcal{X}$ equals 2. This enables us to construct a polynomial-time algorithm for recognizing the Cartan schemes when the rank of G and order of the underlying field are sufficiently large. One of the key points in the proof of the main results is a new sufficient condition for an arbitrary homogeneous coherent configuration to be 2-separable.

The lit-only σ-game and some mathematics around

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This is joint work with Ziqing Xiang from University of Georgia

Let G be a graph. We regard the vertex space, which is the power set of the vertex set of G, as a vector space over the binary field $\mathbb{F}_2$. For each vertex v in G, let T_v be the endomorphism of the vertex space mapping the vertex v to $v + \mathrm{N}_G(v)$, where $\mathrm{N}_G(v)$ is the neighbourhood of v in G, and mapping the vertex w to w itself for all other vertices w in G. In other words, T_v can be written as $\mathrm{id} + \mathrm{N}_G(v)v^*$, where v^* is the Kronecker function for v. Note that T_v is a transvection if v is not a loop vertex, namely if $v \notin \mathrm{N}_G(v)$, while T_v is a projection if v is a loop vertex, namely if $v \in \mathrm{N}_G(v)$.

In the case that G is loopless, the set of all T_v's, where v runs over the vertex set of G, generates a group, called the lit-only group of the graph G. We prove that the lit-only group is a semidirect product of a classical group over $\mathbb{F}_2$ and an elementary abelian 2-group, and we give explicit description of the orbits of the corresponding group action.

In the case that G contains loops, the set of all T_v's, which consists of possible transvections and some projections, generates a monoid. We describe the orbits of this monoid action.

A family of regular coherent non-Schurian graphs, related to extremal graph theory

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This is joint work with Mikhail Klin and Guillermo Pineda-Villavicencio

In this lecture we attempt to establish some natural links between Algebraic and Extremal graph theories (AGT and EGT, respectively).

We start by considering significant concepts in AGT such as coherent configurations and their particular case, association schemes. There exists an efficient, polynomial time, algorithm which for a given graph Γ, calculates the smallest coherent configuration $W(\Gamma)$ containing Γ as a union of basic graphs. Nowadays, $W(\Gamma)$ is called the WL-closure of Γ (in honor of Weisfeiler and Leman). Recently, this subject became more popular due to its links with the graph isomorphism problem.

We call a graph Γ coherent if it is a basic graph of $W(\Gamma)$. The coherent configuration $W(\Gamma)$ is called Schurian if it coincides with the centralizer algebra of $Aut(\Gamma)$, otherwise, $W(\Gamma)$ is non-Schurian.

Many extremal graphs have a rich automorphism group, in particular they are coherent and Schurian. Moore graphs of valencies 3 and 7, as well as the cages which are incidence graphs of classical generalized polygons are examples of such nice objects.

We will try to explain why in the framework of AGT the above mentioned classes of extremal graphs should be naturally substituted by coherent and non-Schurian graphs appearing as a subject of EGT.

A few sporadic examples will be considered together with a family of regular bipartite graphs on $2(q^2-1)$ vertices, $q \geq 3$ is a prime power. These graphs have valency q, diameter 4, and a rank 6 WL-closure with valencies 1, q, $q(q-1)$, $q(q-2)$, $q-1$, $q-2$.

Contributed talks

Spectra in ensembles of regular graphs

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This is joint work with Olga Valba

We consider ensembles of different regular graphs with size distributed in a certain known way. We find the eigenvalue density of such ensembles by analyzing spectra of their adjacency matrices and Laplacian matrices. Such subgraphs as path graphs, full binary and m-ary trees, star-trees are discussed.

The motivation is related to study of macromolecular solutions. It is known that sparse macromolecular clusters can be described by tree ensembles [1].

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Erdös-Ko-Rado Properties of some Finite Groups

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This is joint work with Maryam Jalali-Rad

Let G be a subgroup of the symmetric group $Sym(X)$ and A be a subset of G, where $X = \{1, 2, \ldots, n\}$. The subset A is said to be intersecting if for any pair of permutations $\sigma, \tau \in A$ there is $i \in X$ such that $\sigma(i) = \tau(i)$. A group G has Erdös-Ko-Rado (EKR) property, if the size of any intersecting subset of G is bounded above by the size of a point stabilizer in G. The group G has the strict EKR property if every intersecting set of maximum size is the coset of the stabilizer of a point.

In some recent papers [1–3, 3], the Erdös-Ko-Rado property of $2-$transitive groups and the groups $PGL_2(q), PGL_3(q)$ are investigated. In this talk, we report our recent results on the Erdös-Ko-Rado property of some different classes of finite groups.

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Spectrum and $L-$Spectrum of the Cyclic Graph

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Let G be a finite group. The cyclic graph Γ_G is a simple graph with the vertex set G. Two elements $x, y \in G$ are adjacent in the cyclic graph if and only if $\langle x, y\rangle$ is cyclic [2]. Another graph is the power graph $P(G)$, that was introduced by Kelarev and Quinn in [4]. Two elements $x, y \in G$ are adjacent in the power graph if and only if one is a power of the other. In this paper we continue the work of [1] on computing Laplacian eigenvalues of the power graph of the cyclic and dihedral groups and two unpublished papers [2, 3] in computing eigenvalues of the power graph and its main supergraph for some certain finite groups. As considered application, the algebraic connectivity, the number of spanning trees and Laplacian energy of these graphs were computed for the dihedral, semi-dihedral, cyclic and dicyclic groups.

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Intersection of conjugate solvable subgroups in classical groups of Lie type

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Assume that a finite group G acts on a set Ω. An element $x \in \Omega$ is called a *regular point* if $|xG| = |G|$, i.e. if the stabilizer of x is trivial. Define the action of the group G on Ω^k by the rule

$$g : (i_1, \dots, i_k) \mapsto (i_1 g, \dots, i_k g).$$

If G acts faithfully and transitively on Ω, then the minimal number k such that the set Ω^k contains a G-regular point is called the *base size* of G and is denoted by $b(G)$. For a positive integer m the number of G-regular orbits on Ω^m is denoted by $Reg(G, m)$ (this number equals 0 if $m < b(G)$). If H is a subgroup of G and G acts by the right multiplication on the set Ω of right cosets of H then G/H_G acts faithfully and transitively on the set Ω. (Here $H_G = \cap_{g\in G} H^g$.) In this case, we denote $b(G/H_G)$ and $Reg(G/H_G, m)$ by $b_H(G)$ and $Reg_H(G, m)$ respectively.

Thus $b_H(G)$ is the minimal number k such that there exist elements $x_1, \dots, x_k \in G$ for which $H^{x_1} \cap \dots \cap H^{x_k} = H_G$.

Consider the problem 17.41 from "Kourovka notebook" [1]:

Let H be a solvable subgroup of finite group G and G does not contain nontrivial normal solvable subgroups. Are there always exist five subgroups conjugated with H such that their intersection is trivial?

The problem is reduced to the case when G is almost simple in [2]. Specifically, it is proved that if for each almost simple group G and solvable subgroup H of G condition $Reg_H(G, 5) \geq 5$ holds then for each finite nonsolvable group G and maximal solvable subgroup H of G condition $Reg_H(G, 5) \geq 5$ holds.

Let p be a prime number and $q = p^t$. A cyclic irreducible subgroup $Sin_n(q)$ of $GL_n(q)$ of order $q^n - 1$ is called a *Singer cycle*. If H is a cycle subgroup of $GU_n(q)$ and $|H| = q^n - (-1)^n$ we also call it a Singer cycle and denote by $Sin_n(q)$.

By φ_n we denote an automorphism of $Sin_n(q)$ such that $\varphi_n : g \mapsto g^q$ if $G = GL_n(q)$ and $\varphi_i : g \mapsto g^{q^2}$ if $G = GU_n(q)$.

We have proved the following

Theorem. *Let G be isomorphic to $GL_n(q)$ or $GU_n(q)$ and H be a subgroup of G such that H is block diagonal with blocks isomorphic to $Sin_{n_i}(q) \rtimes \langle\varphi_{n_i}\rangle; i = 1, \dots, k; \sum_{i=1}^{k} n_i = n$. Then $b_H(G) \leq 4$.*

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On finite minimal non-π-closed groups

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Let G be a finite group and π a set of primes. A group G having a normal π-Hall subgroup is called π-closed. We consider groups G such that G is not π-closed and all maximal subgroups of G are π-closed, i.e. G is a minimal non-π-closed group.

Theorem 1 [1, Theorem 1′]. *If G is a minimal non-π-closed group then either $G/\Phi(G)$ is a simple non-abelian group or G is a Schmidt group.*

Thus, the study of the minimal non-π-closed groups reduces practically to the case of the simple non-abelian groups. Further we use the following notation. As usual $\pi(n)$ is the set of all primes dividing a natural n. If q is a prime power then $S(q) := \{q_0 \in \mathbb{N} \mid q = q_0^r$ for a some prime $r\}$. If $P(x)$ is a integral polynomial on x then $\pi_0(P(q)) := \pi(P(q)) \setminus \cup_{q_0 \in S(q)} \pi(P(q_0))$.

Theorem 2 [2, 3]. *Let G be a finite simple non-abelian group different from $PSL_r(q)$ and $PSU_r(q)$ with an odd prime r and $E_8(q)$ (everywhere q is a prime power), and $\pi \subseteq \pi(G)$. The following conditions are equivalent:*

(A) *G is a minimal non-π-closed group;*

(B) *$2 \notin \pi$, $\pi \neq \varnothing$ and one of the following conditions holds:*

(1) *$G \cong A_r$ where $r \geq 5$ is a prime different from $11, 23$ and $(q^n - 1)/(q - 1)$ where q is a prime powers and $n \in \mathbb{N}$, and $\pi = \{r\}$;*

(2) *$G \cong PSL_2(q)$, $q > 5$, $\pi(q) = \{p\}$, and one of the following conditions holds:*

(2a) *$q = p$ and either $\pi \subseteq \pi(p + 1) \setminus \{3, 5\}$ or $p \in \pi \subseteq \pi(p(p^2 - 1)) \setminus \{3, 5\}$;*

(2b) *$q = p^m > p$, $\pi \subseteq \pi_0(q + 1) \setminus \{5\}$, and $3 \notin \pi$ if $p > 2$;*

(3) *$G \cong Sz(q)$ $(q = 2^{2n+1} \geq 8)$, $\pi \subseteq \pi_0(q^2 + 1)$ for non-prime $2n + 1$ and $\pi \subseteq \pi(q^2 + 1)$ for prime $2n + 1$;*

(4) *$G \cong {}^2G_2(q)$ $(q = 3^{2n+1} \geq 27)$, $\pi \subseteq \pi_0(q^2 - q + 1)$ for non-prime $2n + 1$ and $\pi \subseteq \pi(q^2 - q + 1)$ for prime $2n + 1$;*

(5) *$G \cong {}^3D_4(q)$ and $\pi \subseteq \pi_0(q^4 - q^2 + 1)$;*

(6) *$G \cong {}^2F_4(q)$ $(q = 2^{2n+1} \geq 8)$ and $\pi \subseteq \pi_0(q^4 - q^2 + 1)$;*

(7) *G is one of the sporadic groups $M_{23}, J_1, J_4, Ly, Fi_{24}'$, F_2 and π is as in* [2, Theorem 2].

Thus, for the complete description of the all pairs (G, π) where G is a simple minimal non-π-closed group it remains to consider only three series of groups G: $PSL_r(q)$ and $PSU_r(q)$ with an odd prime r and $E_8(q)$.

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Unicyclic signed graphs with minimal energy

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A connected signed graph with n vertices is said to be unicyclic if its number of edges is n. The energy of a signed graph S of order n with eigenvalues $x_1, x_2, \cdots, x_n$ is defined as $E(S) = \sum_{j=1}^{n} |x_j|$. In this paper, we obtain integral representations for the energy of a signed graph. It is shown that even and odd coefficients of the characteristic polynomial of a unicyclic signed graph respectively alternate in sign. As an application of integral representation, energy of signed graphs obtained from a unicyclic graph is compared. As a consequence of these results, we characterize unicyclic signed graphs with minimal energy.

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Completability of hamiltonian cycle in halved cube

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Halved cube $\frac{1}{2}Q_n$ – is a graph, whose vertex set is a set of all binary words of length n with even (odd) weight; two vertices are adjacent if Hamming distance between them equals 2. Halved cube graphs were studied in several papers [1, 2]. We consider hamiltonian cycles in halved cube graphs, in particularly, completability of these cycles. Let

$$C^{1/2} = v_1, v_2, v_3, \dots, v_{2^{n-1}-1}, v_{2^{n-1}}$$

be a hamiltonian cycle in $\frac{1}{2}Q_n$. We call this cycle *completable* if there are binary words $u_1, u_2, \dots u_{2^{n-1}}$ of length n such that cycle

$$C = v_1, \boldsymbol{u_1}, v_2, \boldsymbol{u_2}, v_3, \boldsymbol{u_3}, v_4, \ \dots \ , v_{2^{n-1}}, \boldsymbol{u_{2^{n-1}}}$$

is hamiltonian in Q_n.

For any $n \geq 4$ we prove existence of hamiltonian cycle in $\frac{1}{2}Q_n$, which is not completable.

For hamiltonian cycle $C^{1/2}$ in $\frac{1}{2}Q_n$ we create auxiliary graph $\mathcal{G}\left(C^{1/2}\right)$. Without loss of generality we assume, that $C^{1/2}$ is a cycle, containing all binary words of length n with odd weight.

Vertex set of $\mathcal{G}(C^{1/2})$ – all binary words of length n with even weight. We call vertex $x \in V(\mathcal{G}(C^{1/2}))$ *closest vertex to edge* (u, v) of cycle $C^{1/2}$, if $d(u, v) = d(u, x) + d(x, v)$. Obviously, for any edge of $C^{1/2}$ there are exactly two closest vertices. Then, for every edge e of $C^{1/2}$, we add edge (v, u) to graph $\mathcal{G}$, where u and v are closest vertices to e.

Necessary and sufficient condition of cycle completability is stated in terms of auxiliary graph $\mathcal{G}\left(C^{1/2}\right)$:

Theorem. *Hamiltonian cycle $C^{1/2}$ in $\frac{1}{2}Q_n$ is completable iff there is no trees among connected components of $\mathcal{G}(C^{1/2})$.*

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Groups with the minimal condition for non-abelian noncomplemented subgroups

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Remind that the group G is called Shunkov, if for any its finite subgroup K, every subgroup of the factor group $N_G(K)/K$, generated by two conjugate elements of prime order, is finite (V. D. Mazurov).

The class of periodic Shunkov groups is large and includes, for instance, the classes of 2-groups and binary finite groups. The class of locally graded groups is extremely large. The following new author's theorem holds.

Theorem. *Let G be a non-abelian periodic Shunkov group or a non-abelian locally graded group. Then G satisfies the minimal condition for non-abelian non-complemented subgroups iff it is a Chernikov group or an infinite periodic solvable group with complemented non-abelian subgroups.*

The known Olshanskiy's Examples of infinite simple groups with abelian proper subgroups (see, for instance, [1]) show that in this theorem the condition: "G is periodic Shunkov or locally graded" is essential. Note: the Shunkov groups with the minimal condition for abelian noncomplemented subgroups are completely described by N. S. Chernikov [2].

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Automorphism groups of cyclotomic schemes over finite near-fields

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This is joint work with Andrey Vasil'ev

An algebraic structure $\mathbb{K} = \langle \mathbb{K}, +, \circ \rangle$ is called a (right) *near-field*, if $\mathbb{K}^+ = \langle \mathbb{K}, + \rangle$ is a group, $\mathbb{K}^\times = \langle \mathbb{K} \backslash \{0\}, \circ \rangle$ is a group, $(x+y) \circ z = x \circ z + y \circ z$ for all $x, y, z \in \mathbb{K}$, and $x \circ 0 = 0$ for all $x \in \mathbb{K}$. Finite near-fields can be constructed via finite fields except for a finite number of near-fields [1]. The first ones are called *Dickson* near-fields, the last ones are *Zassenhaus* near-fields.

Let $\mathbb{K}$ be a finite near-field and K be a subgroup of the group $\mathbb{K}^\times$, $\mathcal{R}_K = \{R_K(a) \mid a \in \mathbb{K}\}$, where $R_K(a) = \{(x, y) \in \mathbb{K}^2 \mid x - y \in K \circ a\}$. The pair $(\mathbb{K}, \mathcal{R}_K)$ is called *cyclotomic scheme* over the near-field $\mathbb{K}$ with the base group K. Cyclotomic schemes over finite fields were defined by Delsarte for the algebraic theory of codes [2], cyclotomic schemes over finite near-fields were introduced in [4].

The *automorphism group* of the cyclotomic scheme $\mathcal{C} = (\mathbb{K}, \mathcal{R}_K)$ can be defined as the automorphism group of its partition $\mathcal{R}_K$, namely, $\mathrm{Aut}(\mathcal{C}) = \{g \in \mathrm{Sym}(\mathbb{K}) \mid R^g = R, R \in \mathcal{R}_K\}$. Observe that $\mathrm{Aut}(\mathcal{C}) = \mathrm{Sym}(\mathbb{K})$ if the base group of the cyclotomic scheme $\mathcal{C}$ equals $\mathbb{K}^\times$.

If $\mathbb{F}$ is the finite field of order q, K is a proper subgroup of $\mathbb{F}^\times$, then the automorphism group of the cyclotomic scheme $\mathcal{C} = (\mathbb{K}, \mathcal{R}_K)$ is a subgroup of $\mathrm{A\Gamma L}(1, q) = \{x \mapsto x^\sigma b + c \mid x \in \mathbb{F}, b \in \mathbb{F}^\times, c \in \mathbb{F}^+, \sigma \in \mathrm{Aut}(\mathbb{F})\}$ [3]. The same result was achieved in [4] for cyclotomic schemes over Dickson near-fields with some restrictions on the orders of their base groups. Here we complete the description of the automorphism groups of cyclotomic schemes over finite near-fields.

Theorem. *Let $\mathbb{K}$ be a finite near-field of order q, K a proper subgroup of $\mathbb{K}^\times$, and $\mathcal{C} = (\mathbb{K}, \mathcal{R}_K)$ the corresponding cyclotomic scheme. Then $\mathrm{Aut}(\mathcal{C}) \leq \mathrm{A\Gamma L}(1, q)$ except for a finite number of exceptional schemes. If $\mathcal{C}$ is one of the exceptions, then the subgroup H of $\mathrm{Sym}(\mathbb{K})$ with $\mathrm{Aut}(\mathcal{C}) \leq H$ is determined. In particular, if the base group K is solvable, then so is $\mathrm{Aut}(\mathcal{C})$.*

It is worth mentioning that one of the key tools of our proof is the recent classification of $\frac{3}{2}$-transitive permutation groups [5].

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On periodic subgroups of the finitary linear group over an integral domain

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This is joint work with M. A. Salim

Let $FL_\nu(K)$ be the finitary linear group where K is a ring with the unit, ν is a linearly ordered set. $FL_\nu(K)$ is investigated in [1, 2]. In particular the finitary unitriangular group $UT_\nu(K)$ is studied in [2].

We study periodic subgroups of the finitary linear group $FL_\nu(K)$ in the case where K is an integral domain, ν is a countable set.

The main result of this paper is the theorem.

Theorem. *Let G be a periodic subgroup of $FL_\nu(K)$, K be an integral domain, ν be a countable set. Then G is a (locally nilpotent)-by-countable and locally finite group.*

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Automorphisms of distance-regular graph with intersection array $\{121, 90, 1; 1, 30, 121\}$

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Suppose that Γ — antipodal distance-regular graph of diameter 3 with $\lambda = \mu$, in which the neighbourhood of each vertex is strongly regular with parameters (v', k', λ', μ'). In this case Γ has the intersection array $\{k, \mu(r-1), 1; 1, \mu, k\}$, and the spectrum $k^1, \sqrt{k}^f, -1^k, -\sqrt{k}^f$, where $k = v', \mu = (v'-k'-1)/(r-1)$ and $f = (k+1)(r-1)/2$. Further, the number $(v'+1)(r-1)$ is even. Makhnev A.A. and Samoilenko M.S. [1] selected parameters of strongly regular graphs with no more than 1000 vertices, satisfying these conditions. In this paper automorphisms of distance-regular graph Γ with intersection array $\{121, 90, 1; 1, 30, 121\}$ and of strongly regular graph with parameters $(121, 30, 11, 6)$ are investigated.

Theorem 1. *Let Γ be strongly regular graph with parameters $(121, 30, 11, 6)$, $G = \mathrm{Aut}(\Gamma)$, g element of prime order p of G and $\Omega = \mathrm{Fix}(g)$. Then $\pi(G) \subseteq \{2, 3, 5, 7, 11\}$ and one of the following holds:*

(1) *Ω is empty graph and $p = 11$;*

(2) *Ω is n-clique, either $n = 1$, $p = 3, 5$, or $n = 3t + 1$, $p = 3$ or $n = 2t + 1$ and $p = 2$;*

(3) *Ω is m-coclique, either $m = 3t + 1$, $p = 3$ or $m = 2t + 1$ and $p = 2$;*

(4) *Ω contains an edge and is the union of s isolated cliques, either $p = 3$, the number of vertices in maximal clique from Ω is congruent to 1 by module 3 and s is congruent to 1 by module 3, or $p = 2$, the number of vertices in maximal clique from Ω is odd and s is odd;*

(5) *if Ω contains $[a]$ for some vertex $a \in \Omega$, then $p \leq 3$ and in the case $|\Omega| = 31$ we have $p = 3$;*

(6) *Ω contains geodesic 2-way, $p \leq 7$ and in the case $p = 7$ subgraph Ω is strongly regular with parameters $(16, 9, 4, 6)$.*

Theorem 2. *Let Γ be a distance-regular graph Γ with intersection array $\{121, 90, 1; 1, 30, 121\}$, $G = \mathrm{Aut}(\Gamma)$, g be an element of prime order p of G and $\Omega = \mathrm{Fix}(g)$ contains s vertices in t antipodal classes. Then $\pi(G) \subseteq \{2, 3, 5, 7, 11, 13, 23, 61\}$ and one of the following holds:*

(1) *Ω is empty graph, $p = 2, 61$;*

(2) *Ω is the antipodal class of Γ, $p = 11$;*

(3) *Ω is a t-clique, $p = 3$ and $t = 2, 5, 8, 11$;*

(4) *$p = 23$, Ω is a distance-regular graph with intersection array $\{29, 21, 1; 1, 7, 29\}$, or $p = 13$ and Ω is a distance-regular graph with intersection array $\{17, 12, 1; 1, 4, 17\}$;*

(5) *$p = 7$, $t = 10, 17, 24$ and in the case $t = 10$ subgraph Ω is a distance-regular graph with intersection array $\{9, 6, 1; 1, 2, 9\}$;*

(6) *$p = 5$, $t = 2, 7, 12, 17, 22, 27$ and in the case $t = 7$ subgraph Ω is the union of four isolated 7-cliques;*

(7) *$p = 3, s = 4$, $t = 3l + 2$, $l \leq 9$ and in the case $t = 5$ subgraph Ω is the union of four isolated 5-cliques;*

(8) *$p = 2, s > 0$, any vertex from $\Gamma - \Omega$ is adjacent with even number vertices in Ω and either $s = 2$, $t \leq 60$, or $s = 4$, $t \leq 30$.*

Corollary. *Let Γ be a vertex-symmetric distance-regular graph Γ with intersection array $\{121, 90, 1; 1, 30, 121\}$. Then Γ is the arc-transitive graph with the socle of automorphism group isomorphic to $Z_2 \times L_2(121)$.*

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Threshold function for the edge connectedness of random bipartite graphs

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Random graphs constitute a big area in the modern graph theory; see, e.g., [1]. The Erdős–Rényi model (fixing the number $m(n)$ of the edges) and the Gilbert model (fixing the probability $\rho(n)$ of an edge) are standard models of the random graph. One of the main directions of the study of random graphs concerns threshold functions for different graph properties. Threshold functions are also considered for properties in particular classes of graphs, like bipartite graphs.

In [2], a combinatorial problem on words was reduced to finding the threshold function for the edge connectedness of random bipartite graphs in the Erdős–Rényi model. This function was found in [2] under some restrictions on the size p, q of the parts of the graph (we assume $p \geq q$). If one part is much smaller than the other (namely, $q = o(\frac{p}{\ln p})$), then $\phi(p,q) = \sqrt{pq(\ln q + O(1))}$ is the threshold function; if the sizes of the parts are close ($q = o(\frac{p}{\ln p})$), then the threshold function is $f(p,q) = \frac{pq}{p+q-2}\left(\ln\frac{pq}{p+q-2} + \ln\ln\frac{pq}{p+q-2} + O(1)\right)$.

The above results leave a range of growth rates of q uncovered. We analyze the behaviour of random bipartite graphs in this range. Our contribution is as follows.

1. The problem of searching the threshold function for the edge connectedness of random bipartite graphs in the Erdős–Rényi model is reduced to the same problem in the Gilbert model and vice versa. In particular, $\frac{f(p,q)}{pq}$ and $\frac{\phi(p,q)}{pq}$ are thresholds for the connectedness of random bipartite graphs in the Gilbert model in the same range of growth rates of q.

2. If $q = \alpha\frac{p}{\ln p} + o(\alpha\frac{p}{\ln p})$, $0 < \alpha < \ln\ln p$, then both $\phi(p,q)$ and $f(p,q)$ are strictly smaller than the threshold function; the same result applies for the Gilbert model.

3. If $\alpha < 1$, then $\phi(p,q) > f(p,q)$; replacing $O(1)$ in $\phi(p,q)$ with any $o(\ln q)$ function does not give the threshold function: the expected number of tree components with a single vertex in a smaller part and $(\frac{1}{\sqrt{\alpha}} - \alpha)\ln q$ vertices in a bigger part remains non-zero.

4. If $\alpha > 1$, then $f(p,q) > \phi(p,q)$; replacing $O(1)$ in $f(p,q)$ with any $O(\frac{(\ln\ln q)^2}{\ln\ln\ln\ln\ln q})$ function does not give the threshold function: the expected number of tree components with a single vertex in a smaller part and $2\ln\ln q$ vertices in a bigger part remains non-zero. Moreover, if $\alpha = O(1)$, replacing $O(1)$ in $f(p,q)$ with any $O(\frac{\ln q}{\ln\ln q})$ function does not give the threshold function: the expected number of tree components with a single vertex in a smaller part and $\frac{\ln p}{\ln\ln p}$ vertices in a bigger part remains non-zero.

5. If $\alpha < 1$, both symbolic and numerical computations support the following conjecture: the threshold function grows as $\beta\sqrt{pq\ln q}$, where $\beta > 1$ depends on α and is bounded.

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Asymptotic approximation for the number of n-vertex graphs with given diameter

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Let $\mathcal{J}_{n,\,d=k}$, $\mathcal{J}_{n,\,d\geq k}$, $\mathcal{J}^*_{n,\,d\geq k}$ be the following classes of labeled n-vertex ordinary graphs: graphs of the diameter k, connected graphs of the diameter at least k, and graphs (not necessarily connected) with a shortest path of the length at least k, respectively. It is well known that almost all graphs have diameter 2. Consequently, the number of labeled n-vertex graphs of the diameter 2 is equal to the number $2^{\binom{n}{2}}$ of all n-vertex graphs asymptotically. This result was probably first established in [1]. For the number of graphs of a fixed diameter $k \geq 3$, the asymptotic formula $|\mathcal{J}_{n,\,d=k}| = 2^{\binom{n}{2}}(6\cdot 2^{-k}+o(1))^n$ as $n \to \infty$ was obtained in [2]. The same formula for the number $|\mathcal{J}_{n,\,d\geq k}|$ of connected graphs of the diameter at least k was established in [3]. However, this formula does not give an asymptotically exact value of the number of graphs in the classes $\mathcal{J}_{n,\,d=k}$, $\mathcal{J}_{n,\,d\geq k}$ and error estimates in such asymptotic approximation. For any $k \geq 3$, the asymptotics of the number $|\mathcal{J}_{n,\,d=k}|$ of graphs with the fixed diameter k was found in [4] with an approximation error $r(n)$ satisfying the following estimates for all large enough n: $-c\left(\frac{9}{10}\right)^{n-k} \leq r(n) = O\left(k^2(n-k-1)^4\left(\frac{11}{12}\right)^{n-k-1}\right)$, where $c > 0$ is a constant independent of $n-k-1$.

It is obvious that $\mathcal{J}_{n,\,d=k} \subseteq \mathcal{J}_{n,\,d\geq k} \subseteq \mathcal{J}^*_{n,\,d\geq k}$ and all inclusions are strict for $n \geq k+2$. In the present paper, the asymptotics of the number $|\mathcal{J}^*_{n,\,d\geq k}|$ is found. As a consequence, it is proved that these three classes of graphs have the same asymptotic cardinality. Used methods of graph theory and method of mathematical induction led to a fairly simple proof of calculating the asymptotics of the number $|\mathcal{J}_{n,\,d=k}|$.

Theorem. *Let $k \geq 3$ and $0 < \varepsilon < 1$. Then there is a constant $c_k > 0$ independent of n such that for any $n \in \mathbb{N}$ the following inequalities hold:*

$$2^{\binom{n}{2}}\xi_{n,k}\,(1-\varepsilon_{n,k}) \leq |\mathcal{J}_{n,\,d=k}| \leq |\mathcal{J}_{n,\,d\geq k}| \leq |\mathcal{J}^*_{n,\,d\geq k}| \leq 2^{\binom{n}{2}}\xi_{n,k}\,(1+\varepsilon_{n,k}),$$

$$\textit{where } \xi_{n,k} = q_k\,(n)_{k-1}\left(\frac{3}{2^{k-1}}\right)^{n-k+1},\ \varepsilon_{n,k} = c_k\left(\frac{5+\epsilon}{6}\right)^n = o(1),$$

$$q_k = \frac{1}{2}(k-2)\,2^{-\binom{k-1}{2}},\ (n)_k = n(n-1)\cdots(n-k+1).$$

Note that the asymptotic approximation for the number $|\mathcal{J}_{n,\,d=k}|$ obtained in our theorem is more precise than in [4] if $\varepsilon \in (0, \frac{2}{5})$. Furthermore, the constant c_k is indicated explicitly.

Corollary 1. *Let $k \geq 3$. Then the following asymptotic equalities hold:*

$$|\mathcal{J}_{n,\,d=k}| \sim |\mathcal{J}_{n,\,d\geq k}| \sim |\mathcal{J}^*_{n,\,d\geq k}| \sim 2^{\binom{n}{2}}\xi_{n,k}.$$

Corollary 2. *Almost all graphs of a fixed diameter $k \geq 3$ have a unique pair of diametrical vertices, but almost all graphs of the diameter 2 have more than one pair of such vertices.*

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Structure and Automorphism group of Involution G-Graphs and Cayley Graphs

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This is joint work with A. R. Ashrafi and A. Bretto

Let G be a finite group with identity element 1 and Inv be the set of all involutions in G. The involution $G-$graph $\phi = \phi(G, Inv)$ is a particular $G-$graph introduced by Alain Bretto 2005. The set of vertices of this graph contains the cycles of $(s)x = (x, sx, s^2x, \cdots, s^{o(s)-1}x)$, where $s \in S$ and $x \in T_s$, a set of right transversal of s in G. Two vertices $(s_1)x$ and $(s_2)y$ are adjacent in ϕ when the intersection set of their supports has more than one element. The $G-$graphs have many properties similar to Cayley graphs but they are more general than the Cayley graphs and most of the well-known graphs are $G-$graphs. The aim of this work is obtaining the structures and the automorphism groups of the involution $G-$graphs for some classes of finite groups and some simple groups generated by their involutions and then comparing them with the automorphism groups of the involution Cayley graphs. To do our computational work we use GAP and the finite representation of the groups. Some of the finite groups which we will consider have the representations as follows:

$$\begin{aligned} D_{2n} &= \langle a, b | a^n = b^2 = 1, b^{-1}ab = a^{-1} \rangle, \\ V_{8n} &= \langle a, b | a^{2n} = b^4 = 1, aba = b^{-1}, ab^{-1}a = b \rangle, \\ SD_{8n} &= \langle a, b | a^{4n} = b^2 = 1, bab = a^{2n-1} \rangle, \\ T_{4n} &= \langle a, b | a^{2n} = 1, a^n = b^2, b^{-1}ab = a^{-1} \rangle, \\ U_{2nm} &= \langle a, b | a^{2n} = b^m = 1, aba^{-1} = b^{-1} \rangle. \end{aligned}$$

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On the spectra of non-commuting graphs

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This is joint work with Maryam Jalali-Rad

All graphs considered in this paper are simple and finite. Also, all groups are finite and non-abelian. There are a number of constructions of graphs from groups or semi-groups in the literature. Let G be a non-abelian group with center $Z(G)$. The non-commuting graph (NC-graph) $\Gamma(G)$ is a simple and undirected graph with the vertex set $G \setminus Z(G)$ and two vertices $x, y \in G \setminus Z(G)$ are adjacent whenever $xy \neq yx$. The concept of NC-graphs was first considered by Paul Erdős to answer a question on the size of the cliques of a graph in 1975. For background materials about non-commuting graphs, we encourage the reader to see reference [1]. The non-commuting graph $\Gamma(G)$ of group G was first considered by Paul Erdős to answer a question on the size of the cliques of a graph in 1975, see [2]. In this article, we prove that regular non-commuting graphs are Eulerian. We also prove that there is no $2^s q-$regular non-commuting graph, where q is a prime number greater than 2.

An integral graph is a graph with integral spectrum.

Theorem. *If $\Gamma(G)$ is $k-$regular integral non-commuting graph where $k \leq 16$, then $k = 4$ and $G \cong D_8, Q_8$ or $k = 8$ and $G \cong \mathbb{Z}_2 \times D_8, \mathbb{Z}_2 \times Q_8, SU(2), M_{16}, \mathbb{Z}_4 \ltimes \mathbb{Z}_4, \mathbb{Z}_4 \ltimes \mathbb{Z}_2 \times \mathbb{Z}_2$ or $k = 16$ and $G \cong SmallGroup(32, i)$, where*

$$i \in \{2, 4, 5, 12, 17, 22, 23, 24, 25, 26, 37, 38, 46, 47, 48, 49, 50\}.$$

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Splitting planar graphs of bounded girth to subgraphs with short paths

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A graph $G = (V, E)$ is (a, b)-*partitionable* for positive integers a, b if its vertex set can be partitioned to subsets V_1 and V_2 such that induced subgraphs $G[V_1]$ and $G[V_2]$ do not contain paths of length exceeding $a - 1$ and $b - 1$ respectively. Mihok [4] showed that for any constants a and b their exists series of planar graphs which are not (a, b)-partitionable. However, all examples of graphs constructed by Mihok contain many 3-cycles. On the other hand, for planar graphs with sufficiently large girth it was established in series of papers that they are (a, b)-partitionable for small a and b. For example, in [2] it was proved that any planar graph with girth at least 7 is (2,2)-partitionable. Therefore, a question arises: what is the smallest integer g such that there exist positive integers a, b with the property that any planar graph with girth at least g is (a, b)-partitionable?

For planar graphs with girth at least 6, it was recently proved that they are (5,5)-partitionable [3] and that the vertex set of any such a graph can be partitioned to subsets V_1 and V_2 such that both subgraphs $G[V_1]$ and $G[V_2]$ are linear forests whose paths have length at most 14 [1]. Another important result in [1] is a construction of series of planar graphs with girth 4 which are not (a, b)-partitionable for any given a and b. So it follows by the results in $[1, 3]$, that $5 \leq g \leq 6$.

In this paper we make the final step in determining g by proving that any planar graph with girth at least 5 is (7,7)-partitionable. Hence we establish that $g = 5$. Furthermore, we prove the list version of our main result: if every vertex v of a graph is given a list $L(v)$ of two colours then we can colour the graph vertices from their list in such a way that each monochromatic component is a tree of diameter at most 6.

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Improving some bounds for multiple domination parameters in graphs

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For a graph $G = (V, E)$, the open neighborhood of a vertex $v \in V$ is $N(v) = \{u \in G \mid uv \in E(G)\}$ and the closed neighborhood is $N[v] = N(v) \cup \{v\}$. A set $S \subseteq V$ is a dominating set if each vertex in $V(G) \backslash S$ is adjacent to at least one vertex of S. Equivalently, S is a dominating set of G if $|N(v) \cap D| \geq 1$ for each $v \in V$. Several multiple counterparts of such sets are known. In particular, D is said to be a k-dominating set, if every vertex v not in D satisfies $|N(v) \cap D| \geq k$ or a k-tuple dominating set if $|N[v] \cap D| \geq k$ for each $v \in V$, or a k-tuple total dominating set if every vertex has at least k neighbours in D and etc. We believe all of these concepts can be represented by a comprehensive definition. Therefore we introduce a new domination parameter as a generalization of multiple domination parameters and we improve some results of this topic.

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The linear spectrum of a quadratic APN function and related open problems

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A Boolean function $F : \mathbb{F}_2^n \to \mathbb{F}_2^n$ can be uniquely represented in *algebraic normal form* (ANF) as a n-variable polynomial with coefficients in $\mathbb{F}_2^n$. The *algebraic degree* of F is degree of its ANF. A function F is *affine* (*linear*) if its algebraic degree is not more than 1 (additionally, $F(\mathbf{0}) = \mathbf{0}$); and *quadratic* if its algebraic degree is equal to 2. F and F' are called *extended affine equivalent* (EA-equivalent) if $F' = A' \circ F \circ A'' + A$, where A', A'' are affine permutations of $\mathbb{F}_2^n$ and A is an affine function on $\mathbb{F}_2^n$.

A function F is called *almost perfect nonlinear* (APN) if for any $a, b \in \mathbb{F}_2^n$, $a \neq \mathbf{0}$, equation $F(x)+F(x+a) = b$ has at most 2 solutions. Equivalently, F is APN if $|B_a(F)| = |\{F(x) + F(x+a) \mid x \in \mathbb{F}_2^n\}| = 2^{n-1}$ for any nonzero vector a. APN functions are of a great interest for using in cryptographic applications as S-boxes due to their optimal differential properties. Despite the fact that this class has been intensively studied for about half a century there are many open problems concerning APN functions.

Further we consider only quadratic APN functions. In this case $B_a(F)$ is an affine hyperplane for all nonzero $a \in \mathbb{F}_2^n$ and $B_a(F + L) = B_a(F)$ or $B_a(F + L) = \mathbb{F}_2^n \setminus B_a(F)$, where F is a quadratic APN function and L is a linear function. Let us denote by $k_L^F = |\{a \in \mathbb{F}_2^n \setminus \{\mathbf{0}\} : \ B_a(F) = B_a(F + L)\}|$. By the *linear spectrum* of F we will mean the vector of values $\Lambda^F = (\lambda_0^F, \ldots, \lambda_{2^n-1}^F)$, where λ_k^F is equal to the number of linear functions L such that $k_L^F = k$. It is easy to see that $\sum_{k=0}^{2^n-1} \lambda_k^F = 2^{n^2}$.

Statement 1. *The linear spectrum of a quadratic APN function is a EA-invariant.*

Statement 2. *Let F be a quadratic APN function in n variables, n is even. Then the following statements hold: 1) $\lambda_k^F = 0$ for all even $k = 0, \ldots, 2^n - 1$ and also for all odd $k < (2^n - 1)/3$; 2) $\lambda_{2^n-1}^F \geqslant 2^n$.*

Hypothesis. *Let F be a quadratic APN function in n variables, n is odd. Then the following statements hold: 1) $\lambda_k^F = 0$ for all even $k = 0, \ldots, 2^n - 1$ and also for all odd $k < (2^{n-1} - 1)/3$; 2) $\lambda_{2^n-1}^F = 2^n$.*

This hypothesis is computationally proved for $n = 3, 5$; the second item is also verified for all known quadratic APN functions in 7 variables.

Let us consider two open problems related to the linear spectrum of a quadratic APN function. The first one consists in obtaining an iterative construction of quadratic APN functions mentioned in [1]. For this construction, given a quadratic APN function F in n variables, we need to find a linear function L such that the special admissibility conditions hold for F and L. It can be shown that if $k_L^F > 2^{n-1}$, then these conditions do not hold. The questions arise: what is the minimal k, say k_{min}, such that $\lambda_k^F > 0$ and does there always exist a linear function L with $k_L^F = k_{min}$ such that the admissibility conditions hold?

The second problem consists in finding $\lambda_{2^n-1}^F$ for an arbitrary quadratic APN function F. An answer to this problem will be the first step in solving the wider open problem formulated in [2] by C. Carlet but that was in minds of many specialists. The problem is to describe all APN functions G for a given APN function F such that $B_a(F) = B_a(G)$ for all $a \in \mathbb{F}_2^n$, $a \neq \mathbf{0}$. We prove the following theorem for one known class of quadratic APN functions.

Theorem. *Let $F : \mathbb{F}_{2^n} \to \mathbb{F}_{2^n}$ be a Gold function $F(x) = x^{2^k+1}$, where gcd$(k, n) = 1$. Then the following statements hold: 1) if $n = 4t$ for some t and $k = n/2 \pm 1$, then $\lambda_{2^n-1}^F = 2^{n+n/2}$; 2) otherwise $\lambda_{2^n-1}^F = 2^n$.*

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New construction of Deza graphs

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A graph is called *regular* of valency k, if each of its vertices has exactly k neighbours. A graph is called a *Deza* graph with parameters (n, k, b, a), $b \geq a$, if it has n vertices, is regular of valency k, and the number of common neighbours of any two of its vertices belongs to the set $\{a, b\}$. A Deza graph is called a *strictly Deza* graph, if it has diameter 2 and is not strongly regular.

Let G be a finite group. Let S be a non-empty subset of G such that $1_G \notin S$ and, for any $s \in S$, one also has that $s^{-1} \in S$. A graph $Cay(G, S)$ with the vertex set G and the adjacency defined by $x \sim y \Leftrightarrow xy^{-1} \in S, \ \ \forall x, y \in G$, is called a Cayley graph of the group G with generating set S.

Recall that the Paley graph of order q, where $q \equiv 1 (\mathrm{mod}\ 4)$, is a Cayley graph $Cay(\mathbb{F}_q^+, S_q)$, which is strongly regular with parameters $(n, k, \lambda, \mu) = (q, \frac{q-1}{2}, \frac{q-5}{4}, \frac{q-1}{4})$. Here $\mathbb{F}_q^+$ and S_q are the additive group and the set of non-zero squares of the finite field $\mathbb{F}_q$ of order q, respectively.

Let q_1, q_2 be two odd prime powers such that $q_2 - q_1 = 4$. Let $\overline{S}_{q_1} := \mathbb{F}_{q_1}^* \setminus S_{q_1}$ and $\overline{S}_{q_2} := \mathbb{F}_{q_2}^* \setminus S_{q_2}$ be the sets of non-squares in the corresponding fields. Let $S_0 := \{(0, x) \mid x \in \mathbb{F}_{q_2}^*\}$, $S_1 := S_{q_1} \times \overline{S}_{q_2}$ and $S_2 := \overline{S}_{q_1} \times S_{q_2}$. In this talk we will discuss the following theorem and some related results.

Theorem. *The graph $Cay(\mathbb{F}_{q_1}^+ \times \mathbb{F}_{q_2}^+, S_0 \cup S_1 \cup S_2)$ is a strictly Deza graph with parameters $(v, \frac{v+3}{2}, \frac{v+7}{4}, \frac{v+3}{4})$, where $v = q_1 q_2$.*

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On vertex connectivity of Deza graphs

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We consider finite undirected graphs without loops and multiple edges. A graph is called *regular* of valency k, if each of its vertices has exactly k neighbours. A graph is called a *Deza* graph with parameters (n, k, b, a), $b \geq a$, if it has n vertices, is regular of valency k, and the number of common neighbours of any two of its vertices belongs to the set $\{a, b\}$. A Deza graph is called a *strictly Deza* graph, if it has diameter 2 and is not strongly regular. *The vertex connectivity* $\kappa(\Gamma)$ of a connected graph Γ is the minimum number of vertices one has to remove in order to make the graph Γ disconnected (or empty).

In 1985, Brouwer and Mesner proved [1] that the vertex connectivity of a strongly regular graph is equal to its valency. In 2009, Brouwer and Koolen generalized [2] this result to the class of distance-regular graphs.

Vertex connectivity of strictly Deza graphs obtained from the construction [3, theorem 3.1] based on strongly regular graphs was studied in [3]. The case of the eigenvalue $r \leq 2$ remained open.

In this work we study the vertex connectivity of strictly Deza graphs obtained from strongly regular graphs with eigenvalue $r = 1$ (i.e. from complements to Seidel strongly regular graphs). Note that the construction [3, theorem 3.1] requires the existence of involutive automorphism of a strongly regular graph which interchanges the only non-adjacent vertices. Such automorphisms in the case of the complements to the triangular and lattice graphs have been studied in [2].

In this work the following results were obtained:

Theorem 1. *Let Δ be a strictly Deza graph obtained from either the complement to the triangular graph $T(n)$, $n \geq 3$, or from the complement to one of the following sporadic graphs: Petersen graph, Shrikhande graph, Clebsch graph, Schlafli graph, Chang graph. Then the vertex connectivity of Δ is equal to k, where k is the valency of the graph Δ.*

Theorem 2. *Let Δ be a Deza graph obtained from the complement to $n \times n$-lattice. Then the vertex connectivity of Δ is equal to $k-1$, where k is the valency of the graph Δ.*

Note that in the case of the complement to $n \times n$-lattice, where n is even, and the automorphism which interchanges $n/2$ pairs of rows the construction [3, theorem 3.1] gives vertex-transitive (moreover, Cayley) strictly Deza graphs.

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Hom complex of Mapping cylinders of graphs

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Let $\mathcal{G}$ denote the category whose objects are undirected graphs without multiple edges and morphisms are graph homomorphisms. We will define the notion of double mapping cylinder in the category $\mathcal{G}$. Let $\mathcal{G}'$ be subcategory of $\mathcal{G}$, whose objects do not contain P_3 as an induced subgraph. We will show that the Hom complex functor $Hom(T, \_\_)$ which was defined by Lovász maps double mapping cylinders in graphs to homotopy pushouts in topological spaces where T is a graph in $\mathcal{G}'$.

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A construction of infinite families of directed strongly regular graphs

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The main subject of this talk is directed strongly regular graph, a possible generalization of the well-known (undirected) strongly regular graphs for the directed case, introduced by Duval in [1]. A *directed strongly regular graph* (DSRG) with parameters (n, k, t, λ, μ) is a regular directed graph on n vertices with valency k such that every vertex is incident with t undirected edges; the number of directed paths of length 2 directed from a vertex x to another vertex y is λ if there is an arc from x to y; and μ otherwise.

Using structural analysis of DSRGs we observed that a significant amount of them can be obtained from smaller DSRGs with the aid of suitably defined graph product (π-join) which is based on a partition of the vertex set of the smaller graph. Inspired by this observation we derived the necessary and sufficient conditions to solve, when a homogeneous partition π of a DSRG altogether with π-join construction lead to a bigger DSRG. In fact, the partition π has to be equitable with a prescribed quotient matrix depending just on the parameters of the small DSRG. Using this approach we constructed dozens of infinite families of DSRGs. According to the catalogue of parameter sets with order at most 110, located on the webpage of A. Brouwer and S. Hobart (see [2]), we confirm the existence of DSRGs for 29 open parameter sets.

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Some simple groups which are determined by their character degree graphs

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Let G be a finite group and let $\rho(G)$ be the set of prime divisors of the irreducible character degrees of G. The character degree graph of G, denoted by $\Delta(G)$, is a graph with vertex set $\rho(G)$ and two vertices a and b are incident in $\Delta(G)$, if ab divides some irreducible character degree of G. Many researchers try to know the properties of $\Delta(G)$. For example, in [2] and [3], it was shown that for every group G, diameter of $\Delta(G)$ is at most 3. Also, authors in [3] showed that if G is a finite simple group, then $\Delta(G)$ is connected unless $G \cong PSL(2,q)$. There are many characterizations of finite groups. In [1], Khosravi and et al. introduced a new characterization of finite groups based on the character degree graph as if G has the same order and the character degree graph as that of a certain group M, then $G \cong M$. Khosravi and et al., in [1], proved that the groups of orders less than 6000 are uniquely determined by their character degree graphs and orders. In this talk, we are going to show that some simple groups are uniquely determined by their orders and character degree graphs.

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The four color theorem and Thompson's F

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The four color theorem is known as a theorem whose proof is very long. Bowlin and Brin tried to obtain a shorter proof of the theorem using binary trees, associahedron and an infinite group known as Thompson's F in 2013 [1]. The n-dimensional associahedron is a graph composed by binary trees having $n-2$ leaves. They proved that, if for any pair of vertices D and R in the associahedron, there is a good path called a "valid path" from D to R, then the four color theorem follows. We consider the distance in the n-dimensional associahedron, and proved that for any pair of vertices D and R in the n-dimensional associahedron which have distance n, exists a valid path from D to R. In addition, we found a family $\{G_{n-2,k} \mid 0 \leq k \leq n-2\}$ of vertices in the n-dimensional associahedron such that for every vertex D there is a valid path from D to $G_{n-2,k}$ for some k under a certain assumption. In this talk, we will introduce the relationship between the four color theorem and Thompson's F, and our results.

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Maximum skew energy of tournaments

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We will consider maximum skew energy of tournaments. For a digraph D, the *skew-adjacency matrix* $S(D)$ of D is defined as $S(D) = A(D) - A(D)^T$, where $A(D)$ is the $\{0,1\}$-adjacency matrix of D and *skew energy* $\varepsilon(D)$ is defined as the sum of absolute values of eigenvalues of $S(D)$. A digraph $D = (V, E)$ is called a *tournament* if either $(x, y) \in E$ or $(y, x) \in E$ holds for any pair of $x, y \in V$ $(x \neq y)$.

For any digraph D with n vertices, $\varepsilon(D) \leq n\sqrt{n-1}$ holds [1]. Equality holds if and only if $S(D)$ is a skew symmetric conference matrix. This means that if there exists a digraph D which attains the upper bound, then n is a multiple of 4. Otherwise, we can improve this upper bound. For odd n and any digraph D with n vertices, $\varepsilon(D) \leq (n-1)\sqrt{n}$ holds. Equality holds if and only if D is a doubly regular tournament. Since $n \equiv 3 \pmod 4$ holds if there exist doubly regular tournaments with n vertices [3], there never exists a digraph D which attains the upper bound if $n \equiv 1 \pmod 4$. In both of these upper bounds, tournaments gives the maximum skew energy.

We give the upper bound of skew energy of tournaments with n vertices for $n \equiv 2 \pmod 4$ by using α-skew energy, which is the sum of the α-th power of the absolute values of the eigenvalues of a skew-adjacency matrix. For $n \equiv 2 \pmod 4$ and any tournament T, $\varepsilon(T) \leq 2\sqrt{2n-3} + (n-2)\sqrt{n-3}$ holds.

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On the computational complexity of Roman domination parameters in graph

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A Roman dominating function (or just RDF) on a graph $G = (V, E)$ is a function $f : V \longrightarrow \{0, 1, 2\}$ satisfying the condition that every vertex u for which $f(u) = 0$ is adjacent to at least one vertex v for which $f(v) = 2$. The weight of an RDF f is the value $f(V(G)) = \sum_{u \in V(G)} f(u)$. An RDF f can be represented as $f = (V_0, V_1, V_2)$, where $V_i = \{v \in V : f(v) = i\}$ for $i = 0, 1, 2$. The Roman domination number, $\gamma_R(G)$, of G is the minimum weight of an RDF on G. Several parameters related to the Roman dominating functions have been considered in the very recent years, for example, Roman bondage number, Roman reinforcement number, total Roman domination number, multiple Roman domination number, paired Roman domination number, and etc. We first establish several bounds for the Roman domination number of a graph under some given properties of the graph. We then determine the computational complexity of several Roman domination parameters, and show that the decision problem for these parameters are NP-complete even when restricted to bipartite graphs or chordal graphs. We also study Roman domination parameters in Random graphs.

On the automorphism group of cubic polyhedra

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By using method of graph theory, we can construct cubic graphs whose faces are squares, pentagons and hexagons. We call sometimes these graphs as fullerene graphs, see [1, 2]. In general, a fullerene, is a cubic three connected graph whose faces entirely composed of triangles, squares, pentagones and hexagons. There are many problems concerning with fullerene graphs and many properties of them are studied by mathematician. Given a triple $t = (p_3, p_4, p_5)$, we denote by p_t the class of 3-valent polyhedral graph having p_3 triangular, p_4 quadrilateral, p_5 pentagonal and h hexagonal faces, and no other faces. The number of vertices of a polyhedron belonging to the class p_t is $n = 2p_3 + 2p_4 + 2p_5 + 2h - 4$. In chemical applications it is often important to make the distinction between the symmetry of the polyhedron as a combinatorial object and the physical symmetry of its realization as an affine object in $3D$ space.

In general, a (4,5,6) - polyhedron is a cubic planar graph whose faces are squares, pentagons and hexagons. A (3,5,6) - polyhedron is a cubic planar graph whose faces are triangles, pentagons and hexagons. In this paper, by using the methods of [3], we compute the symmetry group of both (3,5,6) and (4,5,6) polyhedrons.

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On Deza graphs with parameters (v, k, k-1, a)

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By "graph" we mean "an undirected graph without loops and multiple edges". A graph Γ on v vertices is strongly regular with parameters (v, k, λ, μ) if it is regular of degree k, the number of common neighbors of two adjacent vertices is equal to λ and the number of common neighbors of two non-adjacent vertices is equal to μ (see, for example, [1]). A graph Γ on v vertices is a Deza graph with parameters (v, k, b, a), where $v > k \geq b \geq a \geq 0$, if it is regular of degree k and the number of common neighbors of two distinct vertices takes on one of two values a or b, not necessarily depending on the adjacency of the two vertices (see [2]). A strictly Deza graph is a Deza graph which is not strongly regular and has diameter 2.

Let Γ be a strongly regular graph with parameters (v, k, λ, μ). It's not difficult to see, that
(1) if $\mu = k$ then Γ is a complete multipartite graph;
(2) if $\mu = k - 1$ then Γ is the pentagon;
(3) if $\lambda = k - 1$ then Γ is an union of cliques.

Let Γ_1 and Γ_2 be graphs. Γ_2-extension of Γ_1 is a graph obtained by replacing vertices of Γ_1 by copies of Γ_2 and joining all edges between vertices from distinct copies of Γ_2 whenever the correspondent vertices of Γ_1 were adjacent.

In [2] it was obtained a result analogue to (1) for Deza graphs. It was proved, Γ is a strictly Deza graph with parameters (n, k, k, a) if and only if Γ is isomorphic to n_2-coclique extension of a strongly regular graph Γ_1 with parameters (n_1, k_1, λ, μ) for some n_1, k_1, λ, μ and n_2, where $\lambda = \mu$ and $n_2 \geq 2$. Our aim is to obtain results analogue to (2) and (3) for Deza graphs. We prove the following theorem.

Theorem. *A graph Γ is a strictly Deza graph with parameters $(v, k, k-1, a)$ if and only if Γ is isomorphic to 2-clique extension either of a complete multipartite graph or of a strongly regular graph with parameters* $(\frac{v}{2}, \frac{k-1}{2}, \frac{a-2}{2}, \frac{a}{2})$.

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Multiplicities of eigenvalues of the Star graph

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The objects of our research are spectra of Star graphs. The Star graph S_n is the Cayley graph on the symmetric group Sym_n generated by the set of transpositions $\{(1\ 2),(1\ 3),\ldots,(1\ n)\}$. In 2009 A. Abdollahi and E. Vatandoost conjectured [1] that the spectrum of S_n is integral, moreover it contains all integers in the range from $-(n-1)$ up to $n-1$ (with the sole exception that when $n \leqslant 3$, zero is not an eigenvalue of S_n). In 2012 R. Krakovski and B. Mohar [3] proved that the spectrum of S_n is integral, more precisely, they showed that for $n \geqslant 2$ and for each integer $1 \leqslant k \leqslant n$ the values $\pm(n-k)$ are eigenvalues of the Star graph S_n. They also gave a lower bound on multiplicities of eigenvalues of S_n. At the same time, G. Chapuy and V. Feray [2] showed another approach to obtain the exact values of multiplicities of eigenvalues of S_n. Their combinatorial approach is based on the Jucys–Murphy elements and the standard Young tableaux. In 2015 this approach was used to obtain the multiplicities of eigenvalues of S_n for $n \leqslant 10$ [3].

In this talk we present analytic formulas to calculate multiplicities of eigenvalues of the Star graph.

Theorem 1. *Let $n \geqslant 2$ and for each integer $1 \leqslant k \leqslant n$ the values $\pm(n-k)$ are eigenvalues of the Star graph S_n. The multiplicities $mul(n-k)$ for $k = 2,3,4,5$ of eigenvalues of S_n are given by the following formulas:*

$$mul(n-2) = (n-1)(n-2),\ n \geqslant 3; \tag{1}$$

$$mul(n-3) = \frac{(n-3)(n-1)}{2}(n^2-4n+2),\ n \geqslant 4; \tag{2}$$

$$mul(n-4) = \frac{(n-2)(n-1)}{6}(n^4-12n^3+47n^2-62n+12),\ n \geqslant 4; \tag{3}$$

$$mul(n-5) = \frac{(n-2)(n-1)}{24}(n^6-21n^5+169n^4-647n^3+1174n^2-820n+60),\ n \geqslant 5. \tag{4}$$

The following theorem gives an improved lower bound on multiplicity $mul(t)$ of eigenvalues $t := n-k$ of the Star graph which were obtained using the standard Young tableaux.

Theorem 2. *In the Star graph S_n for sufficiently large n and for a fixed t the multiplicity $mul(t)$ of eigenvalue t is at least $2^{\frac{1}{2}n\log n(1-o(1))}$.*

Thus, for any eigenvalue t of S_n the order of logarithm of multiplicities $mul(t)$ is the same that $n!$.

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Computational complexity of Vertex Cover and related problems for highly connected triangulations

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Computational complexity is studied for the Vertex Cover problem (VC) over classes of highly connected plane graphs based on Euclidean distances between their vertices (proximity graphs). Among those graphs are Delaunay triangulations and their special forms, namely half-θ_6 graphs. Study of their graph-theoretic properties received focus in the literature (Dillencourt et. al, 1996, Bose et. al 2012, Biniaz et. al, 2015). Our motivation lies in the field of network security applications as Delaunay triangulations and their relatives represent convenient network topologies which admit efficient local routing (Bose et. al 2014). More specifically VC can be considered as a problem of optimal guarding of a network where one needs to locate positions of sensors (i.e. guards) at the network nodes such that each network link is within the scope of some sensor. Being of interest VC complexity over classes of Delaunay triangulations and their relatives did not get much attention in the literature. In this work we claim VC strong NP-hardness over the class of 4-connected half-θ_6 graphs.

Planar graph is referred to as *planar triangulation* when all faces for some its plane embedding (except for possibly outer face) are triangles. Our approach to study the VC complexity for proximity graphs involves two stages. In the first one problem complexity is studied over specific classes of planar triangulations not taking into account their underlying geometric structure; in the second stage some result from graph drawing is applied that embeds graphs from these classes as Delaunay or half-θ_6 graphs.

It is known that VC is polynomially solvable over classes of outerplanar graphs (Bodlaender, 1998) including outerplanar (always Delaunay realizable due to Dillencourt, 1990) triangulations. Polynomial solvability of VC also holds true over chordal graphs (Gavril, 1972) which become triangulations when they are 3-connected and planar. More general result is known on VC polynomial solvability over class of planar graphs of bounded chordality (Kaminski, 2009). Let us observe that only a few instances of outerplanar and 3-connected planar chordal graphs are in fact 4-connected. The 3-connectivity also requires from instances of the class of planar triangulations whose chordality does not exceed k to have outer facial cycle of length not exceeding k. In our work we claim strong NP-hardness of VC over the class of 4-connected planar triangulations. Considering highly connected instances of triangulations is motivated by the fact that 4-connected (i.e. Hamiltonian) ones are graph isomorphic to Delaunay triangulations (Dillencourt, 1996). Since the proof of this isomorphism is not constructive we get weaker result on the NP-hardness of VC for 4-connected half-θ_6 graphs i.e. Delaunay triangulations build under another metric.

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Finite groups whose prime graphs do not contain triangles

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The prime graph (or the Gruenberg–Kegel graph) $\Gamma(G)$ of a finite group G is a graph, in which the vertices are prime divisors of $|G|$, and two distinct vertices p and q are adjacent if and only if G contains an element of order pq.

Lucido [1] described finite simple groups G such that the connected components of the graph $\Gamma(G)$ are trees, i.e. connected graphs without cycles. Furthermore, in this paper Lucido described the structure of a finite group whose prime graph is a tree. We consider more general problem of the description of the structure of a finite group whose prime graph contains no triangles (3-cycles).

In the talk we discuss both the recent published in [2, 3] and some new our results on this problem.

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Automorphism Group of the Local Fusion Graph of a Finite Group

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Given a finite group G and a conjugacy class X of involutions in G, the local fusion graph $F(G,X)$ has vertex set X and two distinct involutions in X are joined by an edge if their product has odd order. The aim of this paper is to study the automorphism group of the local fusion graphs of dihedral, semi-dihedral, dicyclic, the group $U_{2mn} \cong Z_m \rtimes_{-1} Z_{2n}$ and $V_{8n} = \langle a, b \mid a^{2n} = b^4 = e, aba = b^{-1}, ab^{-1}a = b\rangle$. Some open questions are also presented.

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The structure of Hentzel–Rúa semifield of order 64

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In 1991 G.P. Wene [1] has noted the hypothesis: *any finite semifield D is right or left primitive*, i.e. every element of the loop D^* is a the set of right- or left-ordered powers of an element in a semifield D. In 2004 I. Rúa [2] has indicated a counter-example to Wene's conjecture, using Knuth's semifield of order 32. The second counter-example is a *Hentzel–Rúa semifield* of order 64 [3], which has been constructed in 2007. The Hentzel–Rúa semifield is the unique semifield of order 64 which is neither left nor right primitive.

We investigate the structure of counter-example Hentzel–Rúa semifield and prove the conjecture that its loop is one-generated (weaker than Wene conjecture).

Lemma 1. *The automorphism group of Hentzel–Rúa semifield $\mathcal{H}$ is isomorphic to the symmetric group S_3 and hence has exactly three involution automorphisms.*

Lemma 2. *The semifield $\mathcal{H}$ contains exactly six maximal subfields: 5 subfields of order 8, three from them are stabilizators of different involution automorphisms; the unique subfield of order 4, which is a stabilizator of automorphism of order 3.*

Lemma 3. *The spectrum of the loop $\mathcal{H}^*$ is $\{1, 3, 5, 6, 7\}$. The left and right spectra coincide with $\{1, 3, 6, 7, 12, 15\}$.*

Lemma 4. *For any $n \geq 10$ the loop $\mathcal{H}^*$ is an union of all n-th degrees of any element not from maximal subfields, so $\mathcal{H}^*$ is one-generated.*

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Strongly regular graphs with the same parameters as the symplectic graph

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We consider orbit partitions of groups of automorphisms for the symplectic graph and apply Godsil–McKay switching. As a result, we find four families of strongly regular graphs with the same parameters as the symplectic graphs, including the one discovered by Abiad and Haemers. Also, we prove that switched graphs are non-isomorphic to each other by considering the number of common neighbors of three vertices.

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Induction principle in perfect colorings theory

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Let k be a positive integer. A *k-coloring* of vertices of a graph $G = (V, E)$ is a map $f : V \to \{1, \dots, k\}$. If $f(v) = s$ for some vertex v, then s is the *color* of v. A k-coloring of the vertices of a graph is called *perfect* if the multiset of colors of all neighbors of a vertex depends only on its own color.The induction principle is an efficient approach, that can be used for studying perfect colorings of bipartite graphs.

Consider a bipartite graph $G(V_1, V_2)$. A *half-coloring* of the graph G is a coloring of vertices of the set V_i. A half-coloring is called *feasible*, if it is a part of some perfect coloring of the graph G. Two feasible half-colorings are called *matched*, if they complement each other to the perfect coloring of the whole graph.

Let $f_1 : V_1 \to \{1, \dots, k\}$ be an arbitrary coloring of vertices from the set V_1. *Induction operation* carries on half-coloring f_1 to coloring of the whole graph in the following way: vertices of the set V_2 with the same multiset of neighborhood colors are assigned the same color not from the set $\{1, \dots, k\}$.

The following lemma holds:

Lemma 1. *A half-coloring of any bipartite graph G is feasible if and only if the corresponding induced coloring is perfect.*

A *bipartite coloring* is a perfect coloring of the graph G, if the color sets of the corresponding half-colorings do not intersect. Otherwise the perfect coloring of the graph G is called *nonbipartite*. Note, that color sets of half-colorings coincide in nonbipartite case, if G is connected.

The following concept is proposed to obtain the description of all perfect colorings for the graph G:

1. to obtain the complete description of feasible half-colorings of graph G using Lemma 1;

2. to construct all matched complements for each feasible half-coloring in bipartite and nonbipartite cases.

Let us call the concept described above an *induction principle*.

Authors [1] used the induction principle to obtain the complete description of perfect colorings for the infinite prism graph. It's easy to adapt results obtained in [1] to finite case - finite prism graphs and Mobius ladders.

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Symmetries on Euclidean plane and mechanics

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This is joint work with Pavol Božek and Maria Tothova

The new branches of geometry appear with transferring the results of other mathematical sections, such as set theory, algebra, etc. generally. Hilbert suggested to consider the linguistic rules in addition to the axioms $\mathbb{R}^2$. In the present study hypothesis extended to the plane itself , which is proposed as text. Levels of study of the text are its internal relations. As basic postulates were used: permutation, mirror and with unitary matrix symmetries by Dieudonne, table automorphisms and transfer symmetry by H. Weyl, definition of symmetry by M. Born, binary automorphisms by F. Bachman.

Extended table of Dieudonne symmetries was built on the base of relational algebra and semiotic analyze:

1. Existing of set ($A \neq \emptyset$ Zermelo).
2. Existing of relation ($a_1 \mathbf{R} a_2$ Codd).
3. Membership element of set ($a \in A$ Fraenkel).
4. Universal relation ($f : \Omega \to \Omega'$ implication).
5. Linguistic description of the set (Descartes).
6. Linguistic presentation of the relation (Descartes).
7. Saving cardinality ($\mathbf{m}(A) = const$ Lagrange).
8. Saving power relations ($n = const$ in $C_1x^n + C_2y^n + C_3x^{n-1}y^{n-1} + ... + C_{k-1}x + C_ky + C_0$ Klein).
9. Linguistic order ($\vec{v} = xi + yj + zk + w$ Hamilton).
10. Mathematical order ($a_i \prec a_{i+1}$, where $a_i, a_{i+1} \in \mathbb{R}$ Kantor).
11. Permutation ($a_i \leftrightarrow a_j$).
12. Mirror ($a_i \times -1 = -a_i$).

Symmetries are joined the set theory and universal algebra, so there are two methods for solving the characteristic equation $\mathbf{T}\vec{v} = \lambda\vec{v}$ are exist.

The Jordan curves are the basis for many of the kinematic mechanisms. An exactitude of mechanism causes the trajectory of its motion [2]. The solution of differential equations can be obtained in an analytical form if the trajectory is given of the exact analytical formula. Experiments have shown a difference between the theoretical and actual trajectory less than 5%.

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Hamiltonian Cycles in Directed Toeplitz Graphs

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An $n \times n$ matrix $A = (a_{ij})$ is called a Toeplitz matrix if it has constant values along all diagonals parallel to the main diagonal. A directed Toeplitz graph is a digraph with Toeplitz adjacency matrix. In this talk I will discuss conditions for the existence of hamiltonian cycles in directed Toeplitz graphs.

Notation: The main diagonal of an $n \times n$ Toeplitz adjacency matrix will be labeled 0 and it contains only zeros. The $n-1$ distinct diagonals above the main diagonal will be labeled $1, 2, \ldots, n-1$ and those under the main diagonal will also be labeled $1, 2, \ldots, n-1$. Let $s_1, s_2, \ldots, s_k$ be the upper diagonals containing ones and $t_1, t_2, \ldots, t_l$ be the lower diagonals containing ones, such that $0 < s_1 < s_2 < \cdots < n$ and $0 < t_1 < t_2 < \cdots < n$. Then, the corresponding Toeplitz graph will be denoted by $T_n < s_1, s_2, \ldots, s_k; t_1, t_2, \ldots, t_l >$. That is $T_n < s_1, s_2, \ldots, s_k; t_1, t_2, \ldots, t_l >$ is the graph with vertices $1, 2, \ldots, n$ in which the edge (i, j) occurs if and only if $j - i = s_p$ or $i - j = t_q$ for some p and q $(1 \le p \le k,\ 1 \le q \le l)$.

On the realizability of a finite graph whose complement is triangle-free as the Gruenberg–Kegel graph of a finite group

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By "group" we mean "a finite group" and by "graph" "an undirected graph without loops and multiple edges". Let G be a group. Denote by $\pi(G)$ the set of all prime divisors of the order of G and by $\omega(G)$ the *spectrum* of G, i.e. the set of all its element orders. The set $\omega(G)$ defines the *Gruenberg–Kegel graph* (or the *prime graph*) $\Gamma(G)$ of G; in this graph the vertex set is $\pi(G)$ and distinct vertices p and q are adjacent if and only if $pq \in \omega(G)$.

We say that a graph Γ with $|\pi(G)|$ vertices *is realizable as the Gruenberg–Kegel graph of a group G* if there exists a labeling of the vertices of Γ by distinct primes from $\pi(G)$ such that the labeled graph is equal to $\Gamma(G)$. In [1] it was proved, a graph is realizable as the Gruenberg–Kegel graph of a solvable group if and only if its complement is 3-colorable and triangle free. In [2] it was proved, the Gruenberg–Kegel graph of an almost simple group is isomorphic to the Gruenberg–Kegel graph of an appropriate solvable group if and only if its complement is triangle free. The following question arises.

Question. Is there a graph without 3-cocliques, whose complement is not 3-colorable, but which is isomorphic to the Gruenberg–Kegel graph of an appropriate non-solvable group? In the other words, is there a graph which is realizable as the Gruenberg–Kegel graph of an appropriate non-solvable group, but is not realizable as the Gruenberg–Kegel graph of any solvable group?

The Grotzsch graph is the smallest example of triangle-free graph which is not 3-colorable (see Fig. 1).

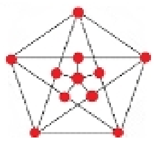

Fig. 1: Grotzsch graph

We prove the following theorem.

Theorem. *The complement of the Grotzsch graph is not realizable as the Gruenberg–Kegel graph of a group.*

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New infinite family of Cameron-Liebler line classes

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This is joint work with Alexander Gavrilyuk and Tim Penttila

Let $\mathrm{PG}(3,q)$ denote the 3-dimensional projective space over the finite field $\mathbb{F}_q$. A *Cameron-Liebler line class* of $\mathrm{PG}(3,q)$ is a set of lines that shares a constant number x of lines with every spread of $\mathrm{PG}(3,q)$. The number x is called the *parameter* of the Cameron-Liebler line class. It can be seen from the definition that the complementary set to a Cameron-Liebler line class is a Cameron-Liebler line class with parameter q^2+1-x, so that we may assume $x \leq (q^2+1)/2$.

The examples of Cameron-Liebler line classes include an empty set of lines ($x=0$), the set of all lines in a plane ($x=1$) or, dually, through a point ($x=1$), and, the union of the previous two examples with $x=1$, assuming that the point is not in the plane ($x=2$). Cameron-Liebler line classes first appeared in the study [1] (see also [8]) on collineation groups of $\mathrm{PG}(n,q)$, $n \geq 3$, that have equally many orbits on lines and on points. For more comprehensive background, we refer to recent papers [3–6].

It was conjectured in [1] that the only Cameron-Liebler line classes are the examples mentioned above, i.e., $x \leq 2$. The first counterexample was found by Drudge in $\mathrm{PG}(3,3)$ with $x=5$, which was generalised later by Bruen and Drudge [2] to an infinite family having parameter $x=(q^2+1)/2$ for all odd q. With the aid of computer Rodgers [7] constructed many more new examples for certain x and prime powers q. Some of them have been shown in [3], [6] to be a part of a new infinite family of Cameron-Liebler line classes with parameter $x=(q^2+1)/2$ for $q \equiv 5$ or $9 \pmod{12}$.

In this work, we construct one more infinite family of Cameron-Liebler line classes in $\mathrm{PG}(3,q)$ with parameter $x=(q^2+1)/2$ for all odd q, which are somehow related to the line classes of Bruen and Drudge, but not equivalent to them. In particular, for $q=5$, there exist at least 3 pairwise non-equivalent Cameron-Liebler line classes with $x=(q^2+1)/2=13$.

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Circulant graphs and Jacobians

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We consider Jacobians of graphs as discrete analogues of Jacobians of Riemann surfaces. More precisely, Jacobian of graph is an Abelian group generated by flows satisfying the first and the second Kirchhoff rules. We define a circulant graph as the Cayley graph of a cyclic group. The family of circulant graphs is quite wide. It includes complete graphs, cyclic graphs, antiprism graphs, even prism graphs and Moebius ladder graph. We propose a new method to find the structure of Jacobians for a large subfamily of circulant graphs.

On 3-generated lattices with standard and dual standard elements among generators

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In [1] a 3-generated lattice is proved to be distributive if two of its generators are standard. It means that such lattice has at most 18 elements. Obviously the same result is true if there are two dual standard generators. We consider 3-generated lattices when one generator is standard and another generator is dual standard.

Theorem. *Let* L *be a* 3*-generated lattice. If one of generator of* L *is standard and another generator is dual standard then* L *contains at most* 21 *elements.*

It should be noted that the estimate in Theorem is sharp.

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On lattices with a neutral pair of elements among generators

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This is joint work with A. G Gein

Studying the structure of a lattice often relies on distinguishing elements with certain good properties, for instance, distributive, standard, or neutral elements of the lattice. For example, a pair of elements a and b is called *distributive* in a lattice L if $c \wedge (a \vee b) = (c \wedge a) \vee (c \wedge b)$ for any element c [1].

We call a pair of elements a and b is called *neutral* in a lattice L if $(x \vee a) \wedge (x \vee b) \wedge (a \vee b) = (x \wedge a) \vee (x \wedge b) \vee (a \wedge b)$ for any element x.

Theorem. *The lattice in Figure is generated by the elements* a, b *and* c *and the pair (*a, b*) is neutral. Conversely, let a lattice be generated by elements* a, b *and* c *such that the pair (*a, b*) is neutral. Then the lattice is a homomorphic image of the lattice in Figure.*

The proof of the theorem is based on the main results of the paper [2].

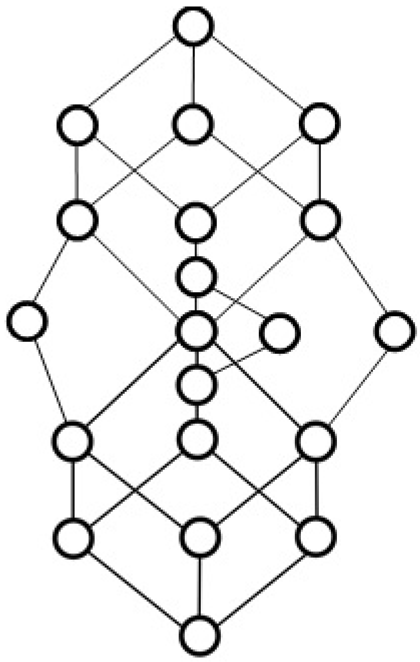

Figure.

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On claw-free strictly Deza graphs

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All graphs under consideration are undirected graphs without loops and multiple edges.

Let $v,\ k,\ b$ and a be integers such that $0 \le a \le b \le k < v$. A graph G is a *Deza graph* with parameters (v, k, b, a) if

- G has exactly v vertices;
- for any vertex u in G its neighbourhood $N(u)$ has exactly k vertices;
- for any two distinct vertices u, w of G the size of $N(u) \cap N(w)$ takes on one of two values b or a.

The key of difference between a strongly regular graph and a Deza graph is that the size of $N(u) \cap N(w)$ does not necessary depend on adjacency u and w.

A *strictly Deza graph* is a Deza graph of diameter 2 that is not strongly regular.

The study of strongly regular graphs has a long history (see for example [1]), and the study of strictly Deza graphs started since the paper written by five authors M. Erickson, S. Fernando, W. H. Haemers, D. Hardy and J. Hemmeter [2] had been published.

A subset of the vertices of a graph is called *coclique* if there does not exist adjacent vertices. The *complete bipartite graph* $K_{m,n}$ is a graph whose set of vertices can be divided into subsets of the cardinalities m and n such that each vertex in one subset is adjacent to every vertex in the other subset and to no vertex in its own subset. A claw is another name for the complete bipartite graph $K_{1,3}$. A *claw-free graph* is a graph that does not have a claw as an induced subgraph. Claw-free graphs were initially studied as a generalization of line graphs. The *line graph* of a graph G is another graph $L(G)$ that represents the adjacencies between edges of G.

In [1] it was described the class of strictly Deza line graphs. In [2] it was described the class of claw-free strictly Deza graphs which are the union of closed neighborhoods of some two non-adjacent vertices, that is there are some two distinct vertices aren't belonging to any 3-coclique.

In this work we proved the following theorem.

Theorem. *Let G be a claw-free strictly Deza graph, and any two of its non-adjacent vertices belong to 3-coclique. Then G is one of that graphs:*

1. *the $4 \times n$-lattice, where $n > 2$, $n \neq 4$;*
2. *the 2-extension of 3×3-lattice;*
3. *Deza line graph with parameters* $(20, 6, 3, 2)$.

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Propelinear codes from multiplicative group of $GF(2^m)$

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All necessary definitions and notions can be found in [1]. It is well known that the *automorphism group* (the isometry group) $\mathrm{Aut}(GF(2^m))$ of the binary vector space $GF(2^m)$ with respect to the Hamming metric is the group of all transformations (x, π) fixing $GF(2^m)$ with respect to the composition $(x, \pi) \cdot (y, \pi') = (x + \pi(y), \pi \circ \pi')$. Given a binary code C the setwise stabilizer of C in $\mathrm{Aut}(GF(2^m))$ is called the *automorphism group* $\mathrm{Aut}(C)$ of C. The *symmetry group* $\mathrm{Sym}(C)$ of a code C is defined as $\mathrm{Sym}(C) = \{\pi \in S_n : \pi(C) = C\}$. A code C is called *transitive* if there is a subgroup H of $\mathrm{Aut}(C)$ acting transitively on the codewords of C. If we additionally require that for any $x, y \in C$, $x \neq y$ there is a unique element h of H such that $h(x) = y$, then H acting on C is called a *regular group* [2] and the code C is called *propelinear* (for the original definition see [3]). In this case the order of H is equal to the size of C. Each regular subgroup $H < \mathrm{Aut}(C)$ naturally induces a group operation on the codewords of C defined in the following way: $x * y := h_x(y)$, such that the codewords of C form a group with respect to the operation $*$, isomorphic to H: $(C, *) \cong H$, which is called *propelinear structure* on C. The notion of propelinearity is important from algebraic and combinatorial coding theory point of view since it provides a general view on linear and additive codes. It is obvious that any propelinear code is transitive but not vice versa. Many known good codes are propelinear, for example all Z_4-linear codes, see also [4] and list of references there.

Theorem 1. *Let $(D, *)$ and $(C, *)$ be propelinear structures such that $(D, *) < (C, *)$ and a group G be a subgroup of $\mathrm{Sym}(C) \cap \mathrm{Sym}(D)$ acting regularly on the right cosets from $(C/D) \setminus D$. Then $C \setminus D$ is propelinear.*

There are many good (uniformly packed and transitive) codes that have the multiplicative group of $GF(2^m)$ as a subgroup of their symmetry group. Taking this group as G we obtain the examples below. Denote by P the Preparata codes constructed in [6]. Denote by H the cyclic Hamming code with the generator polynomial $m_1(x)$ and a Goethals code by Γ. By H', P' and Γ' denote the known Z_4-linear perfect, Preparata and Goethals codes respectively. These codes form nested families: $H \supset P \supset \Gamma$, $H' \supset P' \supset \Gamma'$.

Theorem 2. *Let D be the cyclic code of length n, $n = 2^m$, $m \geq 3$, m is odd, with the generator polynomial $m_1(x)m_{\sigma+1}(x)$, where $((\sigma + 1), (2^m - 1)) = 1$. Then the code $H \setminus D$ is propelinear.*

Theorem 3. *Let n be $4^m, m \geq 2$. The codes $F_2^n \setminus H$, $H \setminus P$, $P \setminus \Gamma$, $H' \setminus P'$ and $P' \setminus \Gamma'$ of length n are propelinear.*

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Affine connections on three-dimensional homogeneous spaces

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Let $(\overline{G}, M)$ be a three-dimensional homogeneous space, where $\overline{G}$ is a Lie group on the manifold M. We fix an arbitrary point $o \in M$ and denote by $G = \overline{G}_o$ the stationary subgroup of o. It is known that the problem of classification of homogeneous spaces $(\overline{G}, M)$ is equivalent to the classification (up to equivalence) of pairs of Lie groups $(\overline{G}, G)$ such that $G \subset \overline{G}$. In the study of homogeneous spaces it is important to consider not the group $\overline{G}$ itself, but its image in $\mathrm{Diff}(M)$. In other words, it is sufficient to consider only effective actions of $\overline{G}$ on M. Since we are interested only the local equivalence problem, we can assume without loss of generality that both $\overline{G}$ and G are connected. Then we can correspond the pair $(\bar{\mathfrak{g}}, \mathfrak{g})$ of Lie algebras to $(\overline{G}, M)$, where $\bar{\mathfrak{g}}$ is the Lie algebra of $\overline{G}$ and $\mathfrak{g}$ is the subalgebra of $\bar{\mathfrak{g}}$ corresponding to the subgroup G. This pair uniquely determines the local structure of $(\overline{G}, M)$, that is two homogeneous spaces are locally isomorphic if and only if the corresponding pairs of Lie algebras are equivalent. A pair $(\bar{\mathfrak{g}}, \mathfrak{g})$ is *effective* if $\mathfrak{g}$ contains no non-zero ideals of $\bar{\mathfrak{g}}$, a homogeneous space $(\overline{G}, M)$ is locally effective if and only if the corresponding pair of Lie algebras is effective. An *isotropic* $\mathfrak{g}$*-module* $\mathfrak{m}$ is the $\mathfrak{g}$-module $\bar{\mathfrak{g}}/\mathfrak{g}$ such that $x.(y+\mathfrak{g}) = [x, y]+\mathfrak{g}$. The corresponding representation $\lambda\colon \mathfrak{g} \to \mathfrak{gl}(\mathfrak{m})$ is called an *isotropic representation* of $(\bar{\mathfrak{g}}, \mathfrak{g})$. The pair $(\bar{\mathfrak{g}}, \mathfrak{g})$ is said to be *isotropy-faithful* if its isotropic representation is injective. If there exists at least one invariant connection on $(\bar{\mathfrak{g}}, \mathfrak{g})$ then this pair is isotropy-faithful [1].

We divide the solution of the problem of classification all three-dimensional isotropically–faithful pairs $(\bar{\mathfrak{g}}, \mathfrak{g})$ into the following parts. We classify (up to isomorphism) faithful three-dimensional $\mathfrak{g}$-modules U, this is equivalent to classifying all subalgebras of $\mathfrak{gl}(3, \mathbb{R})$ viewed up to conjugation. For each obtained $\mathfrak{g}$-module U we classify (up to equivalence) all pairs $(\bar{\mathfrak{g}}, \mathfrak{g})$ such that the $\mathfrak{g}$-modules $\bar{\mathfrak{g}}/\mathfrak{g}$ and U are isomorphic. All of this pairs are described in [2].

Invariant affine connections on $(\overline{G}, M)$ are in one-to-one correspondence [3] with linear mappings $\Lambda\colon \bar{\mathfrak{g}} \to \mathfrak{gl}(\mathfrak{m})$ such that $\Lambda|_{\mathfrak{g}} = \lambda$ and Λ is $\mathfrak{g}$-invariant. We call this mappings *(invariant) affine connections* on the pair $(\bar{\mathfrak{g}}, \mathfrak{g})$. The curvature and torsion tensors of the invariant affine connection Λ are given by the following formulas: $R\colon \mathfrak{m} \wedge \mathfrak{m} \to \mathfrak{gl}(\mathfrak{m})$, $(x_1+\mathfrak{g}) \wedge (x_2+\mathfrak{g}) \mapsto [\Lambda(x_1), \Lambda(x_2)] - \Lambda([x_1, x_2])$; $T\colon \mathfrak{m} \wedge \mathfrak{m} \to \mathfrak{m}$, $(x_1+\mathfrak{g}) \wedge (x_2+\mathfrak{g}) \mapsto \Lambda(x_1)(x_2+\mathfrak{g}) - \Lambda(x_2)(x_1+\mathfrak{g}) - [x_1, x_2]_{\mathfrak{m}}$.

We restate the theorem of Wang on the holonomy algebra of an invariant connection: the Lie algebra of the holonomy group of the invariant connection defined by $\Lambda : \bar{\mathfrak{g}} \to \mathfrak{gl}(3, \mathbb{R})$ on $(\bar{\mathfrak{g}}, \mathfrak{g})$ is given by $V + [\Lambda(\bar{\mathfrak{g}}), V] + [\Lambda(\bar{\mathfrak{g}}), [\Lambda(\bar{\mathfrak{g}}), V]] + \ldots$, where V is the subspace spanned by $\{[\Lambda(x), \Lambda(y)] - \Lambda([x, y]) | x, y \in \bar{\mathfrak{g}}\}$.

We describe all local three-dimensional homogeneous spaces, allowing affine connections, it is equivalent to the description of effective pairs of Lie algebras, and all invariant affine connections on the spaces together with their curvature, torsion tensors and holonomy algebras. We use the algebraic approach for description of connections, methods of the theory of Lie groups, Lie algebras and homogeneous spaces.

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4-colored graphs and complements of knots and links

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A representation for compact 3-manifolds with non-empty non-spherical boundary via 4-colored graphs (i.e. regular 4-valent graphs endowed by a proper edge-coloration with four colors) has been introduced in [1], where an initial tabulation/classification of such manifolds has been obtained, up to 8 vertices of the representing graph.

Computer experiments show that the number of graphs/manifolds grows very rapidly with the increasing of the vertices. As a consequence we focused our attentions on the case of 3-manifolds which are the complements of knots or links in the 3-sphere. In this context we obtained the classification of these 3-manifolds, up to 12 vertices of the representing graphs, showing the type of the links involved (they are exactly 21, and among them 16 are prime).

For the particular case of knot complements, the classification has been recently extended up to 16 vertices: there are exactly 2 knot complements, the trivial knot complement (6 vertices) and the trefoil knot complement (16 vertices).

All these results are contained in [2], which will soon appear on the arXiv.

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A q and q,t-analogue of Hook Immanantal Inequalities and Hadamard Inequality for Trees

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This is joint work with Sivaramakrishnan Sivasubramanian

Let T be a tree on n vertices with Laplacian matrix L and q-Laplacian $\mathcal{L}_q$. Let χ_k be the character of the irreducible representation of the symmetric group $\mathfrak{S}_n$ indexed by the hook partition $k, 1^{n-k}$ and let $\overline{d}_k(L)$ be the normalized hook immanant of L corresponding to the character χ_k. In [1, 3–5], inequalities are known for $\overline{d}_k(L)$ as k increases. By using matchings and assigning statistics to vertex orientations, we generalize these inequalities to the matrix $\mathcal{L}_q$, for all $q \in \mathbb{R}$ and to the bivariate q,t-Laplacian $\mathcal{L}_{q,t}$ for a specific set of values q, t, where both $q, t \in \mathbb{R}$ or both $q, t \in \mathbb{C}$. Our statistic based approach also gives generalizations of inequalities given in [2] for a Hadamard inequality changing index $k(L)$ of L, to the matrices $\mathcal{L}_q$ and $\mathcal{L}_{q,t}$ for trees.

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Some Class of golden graphs and its construction

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In this paper, we generated some class of golden graphs (Graphs with eigenvalues as golden ratio). First, we have proved logically that, for which n (number of vertices), tree A_n (double headed snake) and Prism I_n are golden graphs. Next which Mobious ladder are golden graphs and also proved $C_{5k} + K_1$ are golden graphs. And also for which values of i, j, k the tree $T[i, j, k]$ are golden graph. Similarly, for which values of $i_1, i_2, \ldots, i_n$ the tree $T[i_1, i_2, \ldots, i_n]$ are golden graphs. We have proved logically that the tree U_n (single headed snake) is not golden graph. We have proved the graph $G_1 + G_2$, where G_1 is regular graph and G_2 is prism, as golden graph and also $\overline{K_n} + P_4$ as golden graph. In the end we have constructed golden graphs using prism, Mobious ladder, trees ($T[i, j, k], T[i_1, i_2, \ldots, i_n]$), $C_{5k}+K_1$, $G+P_4$ and $G+C_5$.

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Classifying the fuzzy subgroups of finite symmetric groups

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This is joint work with M. EniOluwafe

The main goal of this paper is to classify the fuzzy subgroups of the finite symmetric group S_n. First, an equivalence relation on the set of all fuzzy subgroups of a group G is defined. Without any equivalence relation on fuzzy subgroups of group G, the number of fuzzy subgroups is infinite, even for the trivial group. In this paper, classifying the fuzzy subgroups structure of a finite symmetric groups $S_n (n \geq 5)$ is made. An explicit formula for the number of distinct fuzzy subgroups of S_n is indicated. We also count the number of fuzzy subgroups for a particular class of finite symmetric groups.

One of the most important problems of fuzzy group theory is to classify the fuzzy subgroups of a finite groups. This topic has enjoyed a rapid development in the last few years. In our case the corresponding equivalence classes of fuzzy subgroups are closely connected to the chains of subgroups in S_n. As a guiding principle in determining the number of these classes, we found the number of maximal chains of S_n. Note that an essential role in solving our counting problem is played again by the Inclusion-Exclusion Principle. It leads us to some recurrence relations, whose solutions have been easily found.

On the average eccentricities of some forbidden subgraphs

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Let G be a connected graph. Let H be a family of connected graphs. Then G is said to be H-free if it contains no induced sub-graph isomorphic to H. There are several important graph classes characterized by a single forbidden induced sub-graph. Examples of graph classes defined in terms of a single forbidden sub-graph is the K_3-free graphs and the C_4-free graphs.

In this talk, I would discussed the average eccentricities of K_3-free graphs and C_4-free graphs in terms of their order and minimum degrees. This can be interpreted as a method of minimizing the average of the maximum time delay of transferring messages from one vertex to the other in a modeled communication network.

The List Distinguishing Number of Graphs

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A graph G is said to be $k-$distinguishable [1] if the vertex set can be colored using k colors such that no nontrivial automorphism of G fixes all the color classes. Distinguishing number $D(G)$ is the least k for which G is $k-$distinguishable.

A graph G is said to be $k-$list distinguishable [3] if each of the vertices can be colored from corresponding given lists of size k such that G is $k-$distinguishable. List distinguishing number $D_l(G)$ is the least k for which G is $k-$list distinguishable. In this talk we discuss some results supporting the conjecture [3] that $D(G) = D_l(G)$ for any graph G. We discuss another statement [2] which strengthen the conjecture [3].

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Φ-Harmonic Functions on Graphs

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Let Φ be a function with some special properties. Properly speaking, it is an N-function. In our talk we will consider a number of aspects of $\Phi-$harmonic analysis on graphs. In particular, we will introduce the key definitions and will reveal that the ones in question are well-defined. Also we will give an overview of our results that bring discrete analogs of classical theorems for harmonic function in the usual sense: uniqueness theorem, Harnack's inequality, Harnack's principle. Our work generalizes results obtained in [1].

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On the number of n-ary quasigroups, Latin hypercubes and MDS codes

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A *latin square* of order n is an $n \times n$ array of n symbols in which each symbol occurs exactly once in each row and in each column. A d-dimensional array with the same property is called a *latin d-cube*. Two latin squares are *orthogonal* if, when they are superimposed, every ordered pair of symbols appears exactly once. If in a set of latin squares, any two latin squares are orthogonal then the set is called *Mutually Orthogonal Latin Squares* (MOLS). From the definition we can ensure that a latin d-cube is the Cayley table of a d-ary quasigroup. Denote by Q the underlying set of the quasigroup. A system consisting of t s-ary functions $f_1, \dots, f_t$ $(t \geq s)$ is *orthogonal*, if for each subsystem $f_{i_1}, \dots, f_{i_s}$ consisting of s functions it holds $\{(f_{i_1}(\overline{x}), \dots, f_{i_s}(\overline{x})) \mid \overline{x} \in Q^s\} = Q^s$. If the system keeps to be orthogonal after substituting any constants for each subset of variables then it is called *strongly orthogonal* (see [2]). If the number of variables equals 2 $(s = 2)$ then such system is equivalent to a set of MOLS. If $s > 2$, it is a set of *Mutually Strong Orthogonal Latin s-Cubes* (MSOLC). A subset C of Q^d is called an *MDS code* (of order $|Q|$ with code distance $t+1$ and with length d) if $|C \cap \Gamma| = 1$ for each t-dimensional face Γ. A system of t MSOLC is equivalent to MDS code with distance $t+1$ (see [2]). Numbers of MOLS, latin d-cubes and MDS codes for small orders are calculated in [4], [7].

Let $N(n, d, \varrho)$ be the number of MDS codes of order n with code distance ϱ and length d. An upper bound $N(n, d, 2) \leq ((1 + o(1))n/e^d)^{n^d}$ is proved in [6].

Theorem. *For each prime number p and $d \leq p+1$ if $3 \leq \varrho \leq p$ or an arbitrary $d \geq 2$ if $\varrho = 2$ it holds $\ln N(p^k, d, \varrho) \geq (k+m)p^{(k-2)m} \ln p(1 + o(1))$ as $k \to \infty$, $m = d - \varrho + 1$.*

Corollary. (a) *The logarithm of the number of latin d-cubes of order n is $\Theta(n^d \ln n)$ as $n \to \infty$.*
(b) *The logarithm of the number of pairs of orthogonal latin squares of order n is $\Theta(n^2 \ln n)$ as $n \to \infty$.*

We use results of [5] to obtain (a) and results of [3] to obtain (b). Item (b) for a subsequence of integers was proved in [1]. Complete text of the report is available in [8].

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On normal edge-transitive Cayley graphs

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This is joint work with Modjtaba Ghorbani

Xu was the first mathematician who proposed the concept of normal Cayley graph in [8] and then Wang et al. [7] obtained all disconnected normal Cayley graphs. Recently, the normality of edge-transitive Cayley graphs is considered by mathematicians and one of the standard problems in this area is to determine the normal edge-transitivity of Cayley graphs with specific orders, see [1, 1, 3, 7]. Baik et al. [1] studied normal edge-transitivity of Cayley graphs on abelian groups of valency at most five and Bosma et al. [2] also considered the edge-transitive Cayley graphs of valency four on non-abelian simple groups. In [2, 9] authors obtained all tetravalent normal edge-transitive Cayley graphs on either a group of odd order or a finite non-abelian simple group. Recently, Kovács [1] classified all connected tetravalent non-normal arc-transitive Cayley graphs on dihedral groups and Darafsheh et al. [3] studied the normal edge-transitive Cayley graphs on non-abelian groups of order $4p$, where p is a prime number. In this paper, we consider the hexavalent normal edge-transitive Cayley graphs on groups of order pqr, where $p > q > r > 2$ are prime numbers.

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Schur rings over elementary abelian two-groups

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Let G be a finite group. Consider the group ring $\mathbb{Z}[G]$. A Schur ring [1, 2] over G is a subring of $\mathbb{Z}[G]$ generated by *simple quantities* $\underline{S}_i$, where $S_i \subseteq G$. Equivalently, a Schur ring over G is an association scheme with G acting regularly as a subgroup of the automorphism group.

Whereas Schur rings over some classes of groups, in particular cyclic groups, have been classified, little is known about other groups. However, recently some progress has been made in terms of enumeration [3]. Elementary abelian groups turned out to be quite resilient.

We report on new results, highlight some of the techniques, and give general constructions of Schur rings over elementary abelian 2-groups.

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On the isomorphism problem for Cayley graphs over abelian p-groups

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Let G be a finite group. A Schur ring over G is a subring of the group ring $\mathbb{Z}G$ that has a linear basis associated with a special partition of G. About 40 years ago Pöschel described all S-rings over cyclic p-groups of odd order. Applying this result Pöschel and Klin solved the isomorphism problem for circulant graphs with p^k vertices, where p is an odd prime.

Let $n = p^{k+1}$, where $p \in \{2, 3\}$ and k is a positive integer. Denote by $\mathcal{G}_n$ and $\mathcal{P}_n$ the class of all graphs on n vertices and the class of graphs on n vertices that isomorphic to Cayley graphs over $G = \mathbb{Z}_p \times \mathbb{Z}_{p^k}$ respectively. Recently all S-rings over G were classified in [1] for $p = 2$ and in [2] for $p = 3$. By using this classification we prove the following theorem.

Theorem. *In the above notation, suppose that the group G is given by its multiplication table. Then the following problems can be solved in time $n^{O(1)}$:*

(1) *given a graph $\Gamma \in \mathcal{G}_n$, test whether $\Gamma \in \mathcal{P}_n$;*

(2) *given graphs $\Gamma, \Gamma' \in \mathcal{P}_n$, test whether $\Gamma \cong \Gamma'$, and (if so) find the set of all isomorphisms between them.*

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On Deza graphs with disconnected second neighbourhoods of vertices

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This is joint work with Sergey Goryainov, Galina Isakova, Vladislav Kabanov and Natalia Maslova

We consider finite undirected graphs without loops and multiple edges.

A graph Γ is called a *Deza graph* if it is regular and the number of common neighbours of two distinct vertices takes on one of two values. A Deza graph Γ is called *strictly Deza graph* if it has diameter 2 and is not strongly regular. Let x be a vertex of a Deza graph Γ. The subgraph of the graph Γ induced by vertices which are at distance 2 from the vertex x is called *second neighbourhood* of the vertex x.

In 1992, Gardiner, Godsil, Hensel, Royle [1] proved that a strongly regular graph which contains a vertex with disconnected second neighbourhood is a complete multipartite graph with parts of the same size greater or equal to 2.

Alexander Gavrilyuk proposed to study strictly Deza graphs which contain a vertex with disconnected second neighbourhood.

Let Γ be a vertex transitive strictly Deza graph such that the second neighbourhood of each its vertex is disconnected. It was proved [2, Theorem 1] that Γ is either edge regular or coedge regular. Also, there were obtained a characterization of a strictly Deza graph which contain a vertex with disconnected second neighbourhood in cases of edge regularity and coedge regularity of this graph.

In this work we proved the following theorem which is a generalization of the previous result [2, Theorem 1].

Theorem. *Let Γ be a strictly Deza graph such that the second neighbourhood of each its vertex is disconnected. Then Γ is either edge regular or coedge regular.*

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About group density function

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Let G be a group and $\mathfrak{N} = \{g_1, ..., g_n\}$ be a set of generators of G. Following [1, p. 102] define

$$F_{(G,\mathfrak{N})}(l) = \begin{cases} 1, \text{ if } l = 0, \\ \text{the number of elements of } G \text{ whose irreducible length in at most l}, \text{ if } l > 0 \end{cases}$$

to be a function of group growth of G on the set $\mathfrak{N}$. Let

$$P_{(G,\mathfrak{N})}(l) = \begin{cases} 1, \text{ if } l = 0, \\ F_{(G,\mathfrak{N})}(l) - F_{(G,\mathfrak{N})}(l-1), \text{ if } l > 0 \end{cases}$$

be a *group density function* of G on the set of generators $\mathfrak{N}$.

Question. Let G and H be groups, $\mathfrak{N}$ and $\mathfrak{M}$ be sets of generators of G and H respectively. Suppose, $P_{(G,\mathfrak{N})}(l) = P_{(H,\mathfrak{M})}(l)$. Are groups G and H isomorphic?

We prove the following theorems.

Theorem 1. *Let G and H be groups, $\mathfrak{N}$ and $\mathfrak{M}$ be sets of generators of G and H respectively. If $P_{(G,\mathfrak{N})}(l) = P_{(H,\mathfrak{M})}(l)$ then $|\mathfrak{N}| = |\mathfrak{M}|$.*

Theorem 2. *Let G and H be groups, $\mathfrak{N}$ and $\mathfrak{M}$ be sets of generators of G and H respectively. If $P_{(G,\mathfrak{N})}(l) = P_{(H,\mathfrak{M})}(l)$ then $|G| = |H|$.*

A set $\mathfrak{N} = \{g_1, ..., g_n\}$ of generators of a group G is called *independent* if $\langle \mathfrak{N} \setminus \{g_i\} \rangle \cap \langle g_i \rangle = \langle e \rangle$ is a trivial subgroup for all $i \in \{1, ..., n\}$.
We prove the following theorem.

Theorem 3. *Let G and H be finite abelian p-groups, $\mathfrak{N}$ and $\mathfrak{M}$ be independent sets of generators of G and H respectively. If $P_{(G,\mathfrak{N})}(l) = P_{(H,\mathfrak{M})}(l)$ then $G \cong H$.*

The most interesting case of the Question is the case when G and H are finite simple non-abelian groups, $\mathfrak{N}$ and $\mathfrak{M}$ are independent sets of their generators.

Conjecture. *Let G and H be finite non-abelian simple groups, $\mathfrak{N}$ and $\mathfrak{M}$ be independent sets of generators of G and H respectively. If $P_{(G,\mathfrak{N})}(l) = P_{(H,\mathfrak{M})}(l)$ then $G \cong H$.*

Theorem 4. *Let $A_8 = \langle \mathfrak{N} \rangle$ and $L_3(4) = \langle \mathfrak{M} \rangle$, where $\mathfrak{N}$ and $\mathfrak{M}$ are independent sets of generators. Then $P_{(A_8,\mathfrak{N})}(l) \neq P_{(L_3(4),\mathfrak{M})}(l)$.*

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Small cycles in the Bubble-Sort graph

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We investigate the Bubble-Sort graph BS_n, $n \geqslant 2$, that is the Cayley graph on the symmetric group Sym_n generated by transpositions from the set $t = \{t_{ii+1} \in Sym_n, 1 \leqslant i \leqslant n-1\}$. In 2006 Yosuke Kikuchia and Toru Arakib have shown [1] that BS_n, $n \geqslant 4$, contains all cycles of even length l, where $4 \leqslant l \leqslant n!$. However a characterization of these cycles has not been done.

In this talk, a characterization of small cycles is given by their canonical forms. A sequence of transpositions $C_l = t_{i_0 i_0+1} ... t_{i_{l-1} i_{l-1}+1}$, where $1 \leqslant i_j \leqslant n-1$, and $i_j \neq i_{j+1}$ for any $j \in \{0, ..., l-1\}$, such that $\pi t_{i_0 i_0+1} ... t_{i_{l-1} i_{l-1}+1} = \pi$, where $\pi \in Sym_n$, is said to be a *form of a cycle* C_l of length l in the Bubble-Sort graph. *The canonical form* C_l of an l-cycle is called a form with a lexicographically maximal sequence of indices. For cycles of a form $C_l = t_{ii+1} t_{jj+1} ... t_{ii+1} t_{jj+1}$, where $l = 2k$, and $t_{ii+1} t_{jj+1}$ appears k times, we write $C_l = (t_{ii+1} t_{jj+1})^k$.

The following results are obtained.

Theorem 1. *Each of the vertices of the Bubble-Sort graph BS_n, $n \geqslant 4$, belongs to $\frac{(n-2)(n-3)}{2}$ different 4-cycles of the canonical form $C_4 = (t_{ii+1} t_{jj+1})^2$, where $1 \leqslant i < j-1 \leqslant n-1$. Totally, there are $\frac{(n-2)(n-3)n!}{8}$ different cycles of length four in the graph.*

Theorem 2. *Each of the vertices of the Bubble-Sort graph BS_n belongs to $(n-2)$ 6-cycles of the canonical form*

$$C_6^1 = (t_{i+1i+2} t_{ii+1})^3, \ \ 1 \leqslant i \leqslant n-2, \ \ n \geqslant 3;$$

to $(n-4)(n-3)$ 6-cycles of the canonical form

$$C_6^2 = (t_{ii+1} t_{i+1i+2})(t_{jj+1})(t_{i+1i+2} t_{ii+1})(t_{jj+1}), 1 \leqslant i < j \leqslant n-1, n \geqslant 5,$$

and to $\frac{(n-3)(n-4)(n-5)}{6}$ 6-cycles of the canonical forms

$$C_6^3 = (t_{ii+1} t_{jj+1} t_{kk+1})^2, k-1 > j > i+1, n \geqslant 6;$$

$$C_6^4 = (t_{ii+1} t_{jj+1} t_{ii+1} t_{kk+1} t_{jj+1} t_{kk+1}), k-1 > j > i+1, n \geqslant 6.$$

In total, there are $\frac{(n^3 - 9n^2 + 29n - 30)n!}{18}$ cycles of length six in the graph.

Analogous results about cycles of length eight are presented by **Theorem 3** in the talk.

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Homotopy type of neighborhood complexes of Kneser graphs

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A. Schrijver identified a family of vertex critical subgraphs of Kneser graphs called the stable Kneser graphs $SG_{n,k}$. A. Bjórner and M. de Longueville proved that the neighborhood complex of the stable Kneser graph $SG_{n,k}$ is homotopy equivalent to a k-sphere. It is also known that the neighborhood complex of $KG_{n,k}$ is homotopy equivalent to the wedge sum of k-spheres. The main objective here is to give the exact number for $KG_{2,k}$, i.e. to show that the homotopy type of the neighborhood complex of $KG_{2,k}$ is a wedge sum of $(k+4)(k+1)+1$ spheres of dimension k. Further we will construct a subgraph $S_{2,k}$ of $KG_{2,k}$ whose neighborhood complex deformation retracts onto the neighborhood complex of $SG_{2,k}$.

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Twisted Edwards curve and its group of points over finite field F_p

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This is joint work with R. V. Skuratovskii

We consider the conditions of supersingularity of Edwards curve [1–3]. A normalization of this curve was constructed by us in projective form. We denote twisted Edwards curve having coefficients a and d as $E_{a,d}$. It was found the mistake in conditions of supersingularity for this curve in theorem 3 of article [4]. More particularly if $p \equiv -3(\bmod 8)$ there is no degenerated twisted pair of curves as it states in [4]. Also if condition $p \equiv \pm 7(\bmod 8)$ holds then the orders of correspondent curves are such $N_{E_2} = N_{E_{2^{-1}}} = p - 3$ that are not equal $p+1$ as it states in [4]. For instance if $p = 31$ then $N_{E_2} = N_{E_{2^{-1}}} = 28 = 8 \cdot 3 + 7 - 3$.

The main result of this paper is the theorem.

Theorem 1. *If $p \equiv 3 \ (mod\ 4)$ and p is prime, then numbers of points on $x^2 + y^2 = 1 + 2x^2y^2$ and on $x^2 + y^2 = 1 + 2^{-1}x^2y^2$ over F_p are equal $N_{E_{1,2}} = N_{E_{1,2^{-1}}} = p + 1$ when $p \equiv 3(\bmod 8)$ and $N_{E_2} = N_{E_{2^{-1}}} = p - 3$ when $p \equiv 7(\bmod 8)$.*

There are two fundamental points [6] $((0, \pm 1), (\pm\sqrt{a}, 0))$ on $E_{a,d}$. The interesting relations between points of $E_{a,d}$ were found.

Theorem 2. *For every no fundamental point (x, y) of $E_{a,d}$ holds the condition $\left(\frac{1-\ ax^2}{p}\right)\left(\frac{1-y^2}{p}\right) = \left(\frac{a-d}{p}\right)$.*

If a is a quadratic residue over F_p then it exists the isomorphism between Edwards curve $E_{1,d}$ and twisted Edwards curve $E_{a,d}$, which is given by the mapping $X \mapsto \sqrt{a}x, Y \mapsto y$. This fact and the theorem 1 lead us to a condition of supersingularity of $E_{a,d}$.

Remark. *Point of order 8 exists on $E_{a,d}$ if and only if point of order 4 exists on $E_{a,d}$ and following conditions holds $(\frac{\frac{1}{d}(1\pm\sqrt{1-\frac{d}{a}})}{p}) = 1$, $(\frac{\frac{a}{d}(1\pm\sqrt{1-\frac{d}{a}})}{p}) = 1$, $(\frac{a}{p}) = 1$, $(\frac{1-\frac{d}{a}}{p}) = 1$.*

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Mathematical Beauty

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Philosophers of mathematical practice have recently returned to the intriguing question of mathematical beauty. This clearly does not belong to traditional aesthetics, because of the abstractness of mathematical matter. Interestingly, mathematicians insist on its presence in mathematics and seem to grant it an important status. They regard mathematical beauty as a major inspiration and often a factor motivating their choices and preferences in practice, such as the search for more elegant proofs and solutions. Philosophers try to find an account for this very special attitude. Sceptics trumpet the fact that mathematics has nothing to please our senses, and claim that all intellectual pleasures are simply epistemic, hence non-aesthetic.

This talk aims to introduce the audience to the recent discussion, present the main actors and the main lines of play. Then it suggests a new angle on the situation from which something new can be learned to defeat a sceptic.

Using a case study from graph theory (the highly symmetric Petersen graph), this talk tries to distinguish genuine aesthetic from epistemic or practical judgements, and correct uses of the word "beautiful" from loose ones. It demonstrates that mathematicians may respond to a combination of perceptual properties of visual representations and mathematical properties of abstract structures; the latter seem to carry greater weight. Mathematical beauty thus primarily involves mathematicians' sensitivity to aesthetics of the abstract.

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On transversals in completely reducible quasigroups and in quasigroups of order 4

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An n-ary operation $f : \Sigma^n \to \Sigma$, where Σ is a set of cardinality q, is called an n-ary quasigroup of order q if in the equality $x_0 = f(x_1, \ldots, x_n)$ knowledge of any n elements of $x_0, x_1, \ldots, x_n$ uniquely specifies the remaining one.

A transversal in a n-ary quasigroup f of order q is a set of $(n+1)$-tuples $\left\{(a_0^i, a_1^i, \ldots, a_n^i)\right\}_{i=1}^q, a_k^i \in \Sigma$ such that $a_0^i = f(a_1^i, \ldots, a_n^i)$ for all $i \in \{1, \ldots, q\}$ and $a_k^i \neq a_k^j$ for all $i \neq j$ and $k \in \{0, \ldots, n\}$.

An n-ary quasigroup f is a composition of an $(n-m+1)$- quasigroup h and an m-quasigroup g if there exists a permutation $\sigma : \{1, \ldots, n\} \to \{1, \ldots, n\}$ such that for all $x_1, \ldots, x_n \in \Sigma$

$$f(x_1, \ldots, x_n) = h(g(x_{\sigma(1)}, \ldots, x_{\sigma(m)}), x_{\sigma(m+1)}, \ldots, x_{\sigma(n)}).$$

An n-ary quasigroup f is called completely reducible if $n \leq 2$ or if it can be represented as a composition of $n-1$ 2-ary quasigroups.

Although there exist completely reducible quasigroups without transversals, we prove that most of such quasigroups do have transversals.

Theorem 1. *Let f be a completely reducible n-ary quasigroup of order q. If n is odd then f has at least $(q \cdot q!)^{\frac{n-1}{2}}$ transversals. If n is even and the most external quasigroup in a decomposition of f has a transversal, then f has at least $(q \cdot q!)^{\lfloor \frac{n-1}{2} \rfloor}$ transversals.*

Also, using a result of [1] we prove the following theorem that sustains a conjecture about transversals in latin hypercubes proposed in [2].

Theorem 2. *If n is odd then every n-ary quasigroup of order 4 has a transversal.*

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Arc-transitive antipodal distance-regular covers of complete graphs: almost simple case

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We consider the problem of classification of arc-transitive antipodal distance-regular graphs of diameter three. Suppose that Γ is such a graph. Then Γ is an antipodal r-cover of K_{k+1}, Γ has intersection array $\{k,(r-1)\mu,1;1,\mu,k\}$, where k is the valency of Γ, r is the size of an antipodal class of Γ and μ denotes the number of common neighbours for any two vertices at distance two in Γ, and each edge of Γ lies in precisely $\lambda = k-(r-1)\mu-1$ triangles. Let Σ be the set of antipodal classes of Γ, let $G = \mathrm{Aut}(\Gamma)$ and let $\bar{G}$ denote the permutation group, induced by G on Σ. Then $\bar{G}$ is 2-transitive on Σ. Thus, the classification of the finite 2-transitive permutation groups is crucial for the study of such graphs, and divides the problem of their description naturally up into two cases: $\bar{G}$ is almost simple or $\bar{G}$ is affine. Our aim is to study the case when the group $\bar{G}$ is almost simple. If $r \in \{2,k\}$, then Γ is distance-transitive and the classification of such graphs can be found in [1]. We also refer to [1, 3] for the case $\lambda = \mu$. Antipodal distance-regular graphs of diameter three that admit an arc-transitive action of $SU_3(q)$ have been recently classified (this result was announced in [4]). We show the following reduction theorem, which states that if $\bar{G}$ is almost simple and $\lambda \neq \mu$, then either Γ is a cover from [4], or $(soc(\bar{G}), k+1) = (L_d(q), (q^d-1)/(q-1))$, where $d \geq 3$.

Theorem. *Suppose Γ is an arc-transitive distance-regular graph with intersection array $\{k,(r-1)\mu,1;1,\mu,k\}$, where $r \notin \{2,k\}$, and $\lambda \neq \mu$. Let $G = \mathrm{Aut}(\Gamma)$, let Σ be the set of antipodal classes of Γ and let $\bar{G}$ denote the permutation group, induced by G on Σ. Suppose further that the socle $\bar{T}$ of the group $\bar{G}$ is a simple non-abelian group, and $(\bar{T}, k+1) \neq (L_d(q), (q^d-1)/(q-1))$, where $d \geq 3$. Then $\bar{T} = U_3(q)$, and $SU_3(q)$ acts arc-transitively on Γ with parameters $k = q^3$ and $\mu = (q+1)(q^2-1)/r$, where r divides $q+1$.*

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On the local structure of Mathon distance-regular graphs

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We study the structure of local graphs of Mathon distance-regular graphs of even valency. This is motivated by the general problem of classification of distance-regular graphs whose local graphs are strongly regular with the second eigenvalue at most a fixed natural number n. We describe several infinite series of locally Δ-graphs of this family, where Δ is a pseudogeometric graph for $pG_l(s,l)$, a strongly regular graph that is a union of some affine polar graphs of type "$-$", or a rank-3 graph, realizable by the van Lint-Shrijver construction.

Theorem. *Let* $q = 2^{2t} > 2$ *and assume that* $r > 1$ *divides* $q-1$. *Let* $\mathrm{M}(q,r)$ *denote a Mathon distance-regular graph with intersection array* $\{q,(r-1)(q-1)/r,1;1,(q-1)/r,q\}$ *and let* Δ *be a local graph of* $\mathrm{M}(q,r)$. *Then* Δ *is arc-transitive and the following assertions hold.*

(1) *If* r *divides* 2^t+1, *then either*

(*i*) $r = 2^t+1$ *and* Δ *is a union of* 2^t *isolated* 2^t*-cliques, or*

(*ii*) $r < 2^t+1$ *and* Δ *is a strongly regular graph with parameters* $(2^{2t},(2^t+1)(2^t-1)/r,((2^t+1)/r-1)((2^t+1)/r-2)+2^t-2,(2^t+1)((2^t+1)/r-1)/r)$.

(2) *If* t *is even and* r *divides* $2^{t/2}+1$, *then either*

(*i*) $r = 2^{t/2}+1$ *and* Δ *is a strongly regular graph with parameters* $(2^{2t},(2^{t/2}-1)(2^t+1),2^{t/2}-2,2^{t/2}(2^{t/2}-1))$ *that is isomorphic to* $\mathrm{VO}^-(4,2^{t/2})$, *or*

(*ii*) $r < 2^{t/2}+1$ *and* Δ *is a strongly regular graph with parameters* $(2^{2t},z(2^{t/2}-1)(2^t+1),z(2^{t/2}-1)(3+z(2^{t/2}-1))-2^t,z(2^{t/2}-1)(1+z(2^{t/2}-1)))$ *which is a union of* $z=(2^{t/2}+1)/r$ *graphs that are isomorphic to* $\mathrm{VO}^-(4,2^{t/2})$.

(3) *If* r *is a prime divisor of* $q-1$, 2 *is a primitive root modulo* r *and* $(r-1)$ *divides* $2t$, *then* Δ *is a rank-3 graph with parameters* $(2^{2t},(2^{2t}-1)/r,(2^{2t}-3r+1+\epsilon(r-1)(r-2)2^t)/r^2,(2^{2t}-r+1-\epsilon(r-2)2^t)/r^2)$ *that is realized by the van Lint–Shrijver construction, where* $\epsilon = (-1)^{2t/(r-1)+1}$.

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Critical phenomena in random graphs

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In first model we consider random nondirected networks subject to dynamics conserving vertex degrees and study, analytically and numerically, equilibrium three-vertex motif distributions in the presence of an external field h coupled to one of the motifs. For small h, the numerics is well described by the chemical kinetics for the concentrations of motifs based on the law of mass action. For larger h, a transition into some trapped motif state occurs in Erdős-Renyi networks.

The second model is devoted to an equilibrium ensemble of large Erdős-Renyi topological random networks with two types of vertices, black and white, and fixed vertex degree prepared randomly with the bond connection probability, p. The system energy is a sum of all unicolor triples (either all black or all white), weighted with a chemical potential of triples, μ. Minimizing the system energy, we see at any positive μ the formation of two predominantly unicolor clusters, linked by a "string"of N_{bw} black-white bonds. The system exhibits a critical behavior manifested in emergence of a wide plateau on the $N_{bw}(\mu)$-curve. We have proposed an explanation of plateau formation in terms of statistical physics, relevant to spinodal decomposition of 1st order phase transitions.

Minimal supports of eigenfunctions of Hamming graphs

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Many combinatorial configurations (for example, perfect codes, latin squares and hypercubes, combinatorial designs and their q-ary generalizations — subspace designs) can be defined as an eigenfunction on a graph with some discrete restrictions. The study of these configurations often leads to the question about the minimum possible difference between two configurations from the same class (it is often related with bounds of the number of different configurations; for example, see [1–5]). Since the symmetric difference of these two configurations is also an eigenfunction, this question is directly related to the minimum cardinality of the support (the set of nonzeros) of an eigenfunction with given eigenvalue. This paper is devoted to the problem of finding the minimum cardinality of the support of eigenfunctions in the Hamming graphs $H(n,q)$. Currently, this problem is solved only for $q = 2$ (see [4]). In [6] Vorob'ev and Krotov proved the lower bound on the cardinality of the support of an eigenfunction of the Hamming graph. In this paper we find the minimum cardinality of the support of eigenfunctions in the Hamming graphs with eigenvalue $n(q-1)-q$ and describe the set of functions with the minimum cardinality of the support.

It is well-known that the set of eigenvalues of the adjacency matrix of $H(n,q)$ is $\{\lambda_m = n(q-1)-qm \mid m = 0,1,\ldots,n\}$. The support of f is denoted by $S(f)$. The set of vertices $x = (x_1, x_2, \ldots, x_n)$ of the graph $H(n,q)$ such that $x_i = k$ is denoted by $T_k(i,n)$. We prove the following theorem:

Theorem. *Let $f : H(n,q) \longrightarrow \mathbb{R}$ be an eigenfunction corresponding to λ_1, $f \not\equiv 0$ and $q > 2$. Then $|S(f)| \geq 2(q-1)q^{n-2}$. Moreover, if $|S(f)| = 2(q-1)q^{n-2}$, then*

$$f(x) = \begin{cases} c, & \text{for } x \in T_k(i,n) \setminus T_m(j,n); \\ -c, & \text{for } x \in T_m(j,n) \setminus T_k(i,n); \\ 0, & \text{otherwise}; \end{cases}$$

where $c \neq 0$ is a constant, i, j, k, m are some numbers and $i \neq j$.

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On Fourier decomposition of Preparata-like codes in the graph of the hypercube

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Given a graph G, a subset $C \subseteq V(G)$ is called a *perfect code (with distance* 3*)* if each vertex of G is at distance no more than 1 from exactly one code vertex. We consider the graph Q^n of n-dimensional binary hypercube. In this case perfect codes exist for every n of form $n = 2^t - 1$ and do not exist for any other n. In the case $n = 4^t - 1$ there exist *Preparata-like codes* (we call them as *Preparata codes*), which are defined as the codes of distance 5 and size $2^{n+1}/(n+1)^2$. Every Preparata code P is included in a unique perfect code $C(P)$ [1]; we denote this perfect code as $C(P)$.

Let f_D^i denotes the orthogonal projection of the characteristic function of the set D to the i-th eigensubspace of the graph of Q^n. (For any function its Fourier transform is defined as the collection of its orthogonal projections to the eigenfunctions $\varphi^\alpha(x) = (-1)^{\langle \alpha, x\rangle}$, $\alpha, x \in Q^n$.)

It is known that for any perfect code C its characteristic function χ_C can be represented as follows:

$$\chi_C = 1/(n+1) + f_C^{(n+1)/2}.$$

Analogously, for an arbitrary Preparata code P we have:

$$\chi_P = 1/(n+1) + f_P^{(n+1)/2} + f_P^k + f_P^h,$$

where $k = (n+1)/2 - \sqrt{n+1}/2$ and $h = (n+1)/2 + \sqrt{n+1}/2$.

Theorem. *Let P be an arbitrary Preparata code in the graph of n-dimensional binary hypercube. Then*

$$f_P^{(n+1)/2} = \frac{2}{n+1} f_{C(P)}^{(n+1)/2}.$$

It is known that an i-component R of an arbitrary Preparata code can be extended to the i-component $S(R)$ of the perfect code $C(P)$. Then we have as a corollary that $f_R^{(n+1)/2} = \frac{2}{n+1} f_{S(R)}^{(n+1)/2}$.

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On products of submodular subgroups of finite groups

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Throughout this report, all groups are finite. Recall that a subgroup M of a group G is said to be modular in G if M is a modular element of the lattice of all subgroups of G [1]. It means that the following conditions are fulfilled:

(1) $\langle X, M \cap Z\rangle = \langle X, M\rangle \cap Z$ for all $X \leq G, Z \leq G$ such that $X \leq Z$, and

(2) $\langle M, Y \cap Z\rangle = \langle M, Y\rangle \cap Z$ for all $Y \leq G, Z \leq G$ such that $M \leq Z$.

In the paper [2] I. Zimmermann introduced the notion of a submodular subgroup which generalizes the notion of a subnormal subgroup. Recall that a subgroup H of a group G is said to be submodular in G [2], if there exists a chain of subgroups $H = H_0 \leq H_1 \leq \ldots \leq H_{s-1} \leq H_s = G$ such that H_{i-1} is a modular subgroup in H_i for $i = 1, \ldots, s$.

In [3] the class $sm\mathfrak{U}$ of all groups with submodular Sylow subgroups was investigated and some of its properties were found. For instance, it was proved in [3] that $sm\mathfrak{U}$ forms a hereditary saturated formation, its local function was found, criteria of the membership of a group to the class $sm\mathfrak{U}$ were established.

This report is devoted to the further development of results of the paper [3]. In particular, we obtained the following result.

Theorem. *Let G be a group, $G = G_1G_2$ be a product of submodular subgroups G_1 and G_2 such that $G_i \in sm\mathfrak{U}$, $i = 1, 2$, and $(|G : G_1|, |G : G_2|) = 1$. Then $G \in sm\mathfrak{U}$.*

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Majorana Representations of Triangle-Point Groups

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Majorana theory was introduced by A. A. Ivanov [1] in 2009 as the axiomatization of certain properties of the 2A-axial vectors of the 196,884-dimensional Griess algebra. Ivanov's work was inspired by a result of S. Sakuma [2] which reproved certain important properties of the Griess algebra in the context of vertex operator algebras. Majorana theory takes the key hypotheses of Sakuma's result to provide a powerful framework, independent of vertex operator algebras, in which to study the Griess algebra and other related objects.

The axioms of Majorana theory can be used to define and construct objects known as Majorana algebras and Majorana representations. In this talk, I will present my own work on Majorana representations of triangle-point groups. A triangle-point group is a group G which is generated by three involutions a, b and c such that a and b commute and such that the product of any two elements of $T := a^G \cup b^G \cup c^G \cup (ab)^G$ has order at most 6. They play an important role in the study of the Monster group and the Monster graph.

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A criterion of unbalance of some simple groups of Lie type

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A finite simple nonabelian group K is called locally balanced (locally 1-balanced) with respect to a prime p if $O_{p'}(C_G(x))$=1 for any element x of order p from $G = Aut(K)$. The locally unbalanced finite simple nonabelian groups were determined in Theorem 7.7.1 from [1]. However, the item (d) of this theorem is wrong. This mistake is removed by the following theorem.

Theorem. *Let G be a finite almost simple group, $K = Soc(G)$ be a group of Lie type over a field of characteristic r, and $x \in G \setminus Inndiag(K)$ be an element of a prime order $p \neq r$. Then the following conditions are equivalent:*

(1) $O_{p'}(C_G(x)) \neq 1$;

(2) x induces a field automorphism on K and $(|C_K(x)|, p) = 1$.

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On Eulerian Identities in $UT_n(\mathbb{Q})$

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One of the known methods of building identities in matrix rings deals with eulerian multigraphs. A multigraph is a graph which is permitted to have multiple oriented edges with same end nodes, and loops - edges with same start and end node. Suppose, $\mathcal{G}$ is an eulerian graph with n edges and $\Pi(\mathcal{G})$ is the set of all its eulerian paths. Every such path π we consider to be a permutation of set $\{1, 2, \ldots, n\}$, by $sgn(\pi)$ we denote the sign of this transposition. An *eulerian polynomial of the graph* $\mathcal{G}$ is the polynomial:

$$P_{\mathcal{G}} = \sum_{\pi \in \Pi(\mathcal{G})} sgn(\pi) x_{\pi(1)} \cdots x_{\pi(n)}.$$

The examples of eulerian polynomials are well known standard polynomials

$$\mathcal{S}_n = \sum_{\pi \in S_n} sgn(\pi) x_{\pi(1)} \cdots x_{\pi(n)},$$

which are built from graphs with one vertex and n loops. The identity $f = 0$ is called eulerian, if f is an eulerian polynomial of some graph.

The identity basis of 2×2 matrices ring from eulerian polynomials is already found by M. Domokos in [1]. The following result deals with ring of upper triangular matrices $UT_n(\mathbb{Q})$, the identity basis of such ring from non-eulerian polynomials is also found by Yu. N. Maltsev in [2].

Theorem. *In the class of rings with 1, the eulerian polynomial*

$$P_{UT_n}(x_1, \ldots, x_n, y_1, \ldots, y_n, e_1, \ldots, e_{n-1}) = [x_1, y_1]e_1[x_2, y_2]e_2 \ldots e_{n-1}[x_n, y_n]$$

forms basis of the ring $UT_n(\mathbb{Q})$. The polynomial P_{UT_n} is built from graph G_{UT_n} displayed on fig.1.

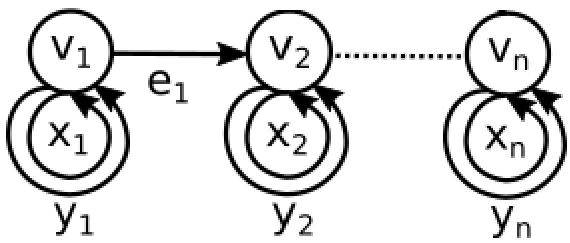

Рис. 1: G_{UT_n}

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On the spectra of automorphic extensions of finite simple exceptional groups of Lie type

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Given a finite group G, the *spectrum* of G is the set of orders of its elements. This set is denoted by $\omega(G)$. Groups are called *isospectral* if their spectra are equal. The number of pairwise non-isomorphic groups isospectral to a group G is denoted by $h(G)$. A group G is called *recognizable by spectrum* if $h(G) = 1$, and *almost recognizable* if $h(G)$ is finite. We say that the *recognition problem* is solved for a group G, if $h(G)$ is known, and if $h(G)$ is finite, the description of all pairwise non-isomorphic groups isospectral to G is given.

Let S be a finite simple exceptional group of Lie type and $S \neq {}^3D_4(2)$. Then S is known to be almost recognizable. Moreover, every finite group isospectral to S is isomorphic to some group G such that $S \leq G \leq \mathrm{Aut}(S)$ [1, Theorem 1]. So, in order to solve the recognition problem for a group S, it suffices to describe all groups G such that $\omega(G) = \omega(S)$ and $S \leq G \leq \mathrm{Aut}(S)$. Such descriptions already exist for all exceptional groups except for the groups of types E_6 and E_7. Let us denote groups of type E_6 by $E_6^{\varepsilon}(q)$, $\varepsilon \in \{+, -\}$, where $E_6(q) = E_6^{+}(q)$ and ${}^2E_6(q) = E_6^{-}(q)$. We complete the investigation of recognition problem for finite simple exceptional groups of Lie type by following results:

Theorem 1. *Let S be a finite simple exceptional group $E_6^{\varepsilon}(q)$, where q is a power of prime p, and let $S < G \leq \mathrm{Aut}(S)$. Then $\omega(G) = \omega(S)$ if and only if G is an extension of S by a field automorphism, G/S is a 3-group, 3 divides $q - \varepsilon 1$ and $p \notin \{2, 11\}$.*

Theorem 2. *Let S be a finite simple exceptional group $E_7(q)$, where q is a power of prime p, and let $S < G \leq \mathrm{Aut}(S)$. Then $\omega(G) = \omega(S)$ if and only if G is an extension of S by a field automorphism, G/S is a 2-group and $p \notin \{2, 13, 17\}$.*

These results also complete the study of recognition problem for finite simple groups of Lie type over fields of characteristic 2 (results for classical groups are given in [2]).

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The International Conference and PhD-Master Summer School «Graphs and Groups, Spectra and Symmetries» 2016

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	10:00 – 13:00 Morning Sessions					
	PhD-Master Summer School		Conference	PhD-Master Summer School		Conference
	10:00 – 10:50 Minicourse 1: Hsu Lecture 1	10:00 – 10:50 Minicourse 1: Hsu Lecture 3	10:00 – 10:50 Invited talk: Marušič	10:00 – 10:50 Minicourse 1: Hsu Lecture 5	10:00 – 10:50 Minicourse 1: Hsu Lecture 7	10:00 – 10:50 Invited talk: Munemasa
	11:00 – 11:50 Minicourse 1: Hsu Lecture 2	11:00 – 11:50 Minicourse 1: Hsu Lecture 4	11:00 – 11:50 Invited talk: Kutnar	11:00 – 11:50 Minicourse 1: Hsu Lecture 6	11:00 – 11:50 Minicourse 1: Hsu Lecture 8	11:00 – 11:50 Invited talk: Östergård
	11:50 – 12:10 Coffee break					
	12:10 – 13:00 Minicourse 2: Dobson Lecture 1	12:10 – 13:00 Minicourse 2: Dobson Lecture 3	12:10 – 13:00 Invited talk: Muzychuk	12:10 – 13:00 Minicourse 2: Dobson Lecture 5	12:10 – 13:00 Minicourse 2: Dobson Lecture 7	12:10 – 13:00 Invited talk: Kazarin
	13:00 – 14:30 Lunch					
	14:30 – 19:00 Afternoon Sessions					
	PhD-Master Summer School		Conference	PhD -Master Summer School		Conference
	14:30 – 15:20 Minicourse 2: Dobson Lecture 2	14:30 – 15:20 Minicourse 2: Dobson Lecture 4	14:30 – 15:20 Invited talk: Wu	14:30 – 15:20 Minicourse 2: Dobson Lecture 6	14:30 – 15:20 Minicourse 2: Dobson Lecture 8	14:30 – 15:20 Invited talk: Vasil`ev
	15:20 – 16:00 Coffee break					
19:00 – 21:00 Welcome party Hall of the Conference room 3307 Novosibirsk State University	16:00 – 19:00 Conference					Special Session
	Goryainov	Kondrat`ev	16:00 – 16:50 Invited talk: T.Ito	Gholaminezhad	Kubota	16:00 – 17:00 Ponomarenko "Graph isomorphism in quasipolynomial time" (L. Babai, 2015)
	Shalaginov	Maslova		RajabiParsa	Mogilnykh	
	Kabanov	Zvezdina	17:00 – 17:50 Invited talk: Koolen	Narayan	Gorodilova	
	Mityanina	Zenkov		Simonova	Potapov	
	17:50 – 18:10 Coffee break					
	Tsiovkina	Baykalov	18:10 – 19:00 Invited talk: Gavrilyuk	Golmohammadi	Vasil`eva	17:00 – 19:00 Discussions
	Gyürki	Ghorbani		Miningulov	Taranenko	
	19:00 – 20:00 Dinner					19:00 – 22:00 Conference dinner
	20:00 – 22:00 Problem solving Minicourse 1	20:00 – 22:00 Problem solving Minicourse 2	Football/ Volleyball NSU Sports Complex	20:00 – 22:00 Problem solving Minicourse 1	20:00 – 22:00 Problem solving Minicourse 2	

The International Conference and PhD-Master Summer School «Graphs and Groups, Spectra and Symmetries» 2016

Monday August 22	Tuesday August 23	Wednesday August 24	Thursday August 25	Friday August 26	Saturday August 27	Sunday August 28
Excursions/ Sport Activities	07:30 – 09:45 Breakfast					
	10:00 – 13:00 Morning Sessions					
	PhD-Master Summer School		Conference	PhD-Master Summer School		Conference
	10:00 – 10:50 Minicourse 3: Ivanov Lecture 1	10:00 – 10:50 Minicourse 3: Ivanov Lecture 3	10:00 – 10:50 Invited talk: Betten	10:00 – 10:50 Minicourse 3: Ivanov Lecture 5	10:00 – 10:50 Minicourse 3: Ivanov Lecture 7	10:00 – 10:50 Invited talk: Hirasaka
	11:00 – 11:50 Minicourse 3: Ivanov Lecture 2	11:00 – 11:50 Minicourse 3: Ivanov Lecture 4	11:00 – 11:50 Invited talk: Tarannikov	11:00 – 11:50 Minicourse 3: Ivanov Lecture 6	11:00 – 11:50 Minicourse 3: Ivanov Lecture 8	11:00 – 11:50 Invited talk: Ham
	11:50 – 12:10 Coffee break					
	12:10 – 13:00 Minicourse 4: Mohar Lecture 1	12:10 – 13:00 Minicourse 4: Mohar Lecture 3	12:10 – 13:00 Invited talk: Kwak	12:10 – 13:00 Minicourse 4: Mohar Lecture 5	12:10 – 13:00 Minicourse 4: Mohar Lecture 7	12:10 – 13:00 Invited talk: Nedela
	13:00 – 14:30 Lunch					Closing
	14:30 – 19:00 Afternoon Sessions					
	PhD-Master Summer School		Conference	PhD-Master Summer School		
	14:30 – 15:20 Minicourse 4: Mohar Lecture 2	14:30 – 15:20 Minicourse 4: Mohar Lecture 4	14:30 – 15:20 Invited talk: Du	14:30 – 15:20 Minicourse 4: Mohar Lecture 6	14:30 – 15:20 Minicourse 4: Mohar Lecture 8	
	15:20 – 16:00 Coffee break					
	16:00 – 19:00 Conference					
	Whybrow	Goyal	16:00 – 16:50 Invited talk: Ponomarenko	Ryabov	Inoue	
	Churikov	Nagar		Reichard	Glebov	
	Kravtsova	Jalali-Rad	17:00 – 17:50 Invited talk: Ziv-Av	Valyuzhenich	Bykov	
	Matkin	Panenko		Mednykh	Osaye	
	17:50 – 18:10 Coffee break					
	Skuratovskii	K.Ito	Open problems session	Andryukhina	Heydari	
	Shlepkin	Mulazzani		Kobylkin	Starikova	
	19:00 – 20:00 Dinner					
	20:00 – 22:00 Problem solving Minicourse 3	20:00 – 22:00 Problem solving Minicourse 4	Football/ Volleyball NSU Sports Complex	20:00 – 22:00 Problem solving Minicourse 3	20:00 – 22:00 Problem solving Minicourse 4	

Groups and Graphs, Metrics and Manifolds

Yekaterinburg, Russia, July, 22–30, 2017

Announcement

Krasovskii Institute of Mathematics and Mechanics of Ural Branch of Russian Academy of Sciences, Ural Federal University named after the first President of Russia B. N. Yeltsin and Chelyabinsk State University organize the International Conference and PhD-Master Summer School "Groups and Graphs, Metrics and Manifolds" (G2M2). All scientific activities will take place in one of the recreation areas near Yekaterinburg, Russia, July, 22–30, 2017.

G2M2 aims to cover modern aspects of group theory, graph theory and 3-manifold topology, including knot theory.

The scientific program of G2M2 includes:

- Lectures of main speakers
- Short contributions in sections
- Minicourses in the frame of the PhD-Master Summer School

The official language of G2M2 is English.

Confirmed Lecturers:

Michel-Marie Deza
Ecole Normale Superieure, Paris, France

Olga G. Kharlampovich
Department of Mathematics and Statistics, Hunter College, CUNY, New York, USA

Mikhail V. Volkov
Ural Federal University, Yekaterinburg, Russia

The official website of G2M2 is http://g2m2.imm.uran.ru.